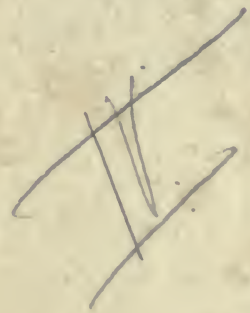
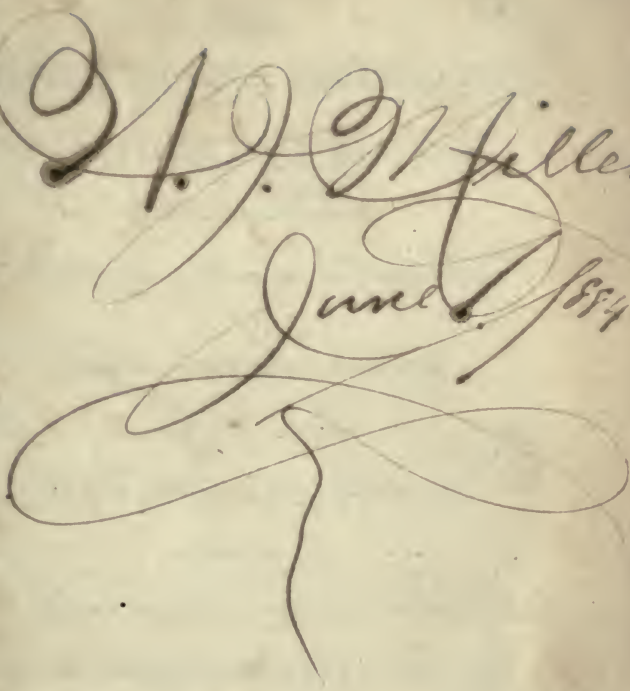
UC-NRLF
$B 306 329



LIBRARY
OF THE
UNIVERSITY
OF
CALIFORNIA

EDUCATION LIBR.



THE

UNITED STATES CALCULATOR;

OR, ARITHMETIC SIMPLIFIED,

IN DOLLARS AND CENTS,

ADAPTED TO THE COMMERCE OF THE UNITED STATES,

IN ITS

FOREIGN AND DOMESTIC RELATIONS,

ABBREVIATED, SIMPLIFIED, AND ARRANGED ON A NEW SYSTEM,

IN A SERIES OF LECTURES,

COMPRISING THE

ANALYTICAL AND SYNTHETICAL METHODS OF DEMONSTRATION,

DESIGNED FOR SCHOOLS AND ACADEMIES.

BY JOHN M'NEVIN,
TEACHER OF ENGLISH AND MATHEMATICS

BALTIMORE:
PUBLISHED BY FIELDING LUCAS, JR.
No. 138 *Market street.*

PRINTED BY JAMES YOUNG,
1841.

Price One Dollar.

Entered according to Act of Congress, in the year 1840, by JOHN M'NEVIN, in the Clerk's Office, of the District Court of Maryland.

EDUCATION LIBR.

QA 102
M17
Educ.
Lib.

CONTENTS.

Page.

LECTURE I.

Numeration.....12
Preliminary Remarks.....13

LECTURE II.

Addition.....18

LECTURE III.

Multiplication.....34
Mental Operations.....37
Calculation of Bills.....46

LECTURE IV.

Subtraction.....54

LECTURE V.

Division.....61
Properties of Numbers.....79
Analysis of Numbers.....84

LECTURE VI.

Decimals.....90
Addition of Decimals.....93
Multiplication of Decimals.....94
Subtraction of Decimals.....96
Division of Decimals.....97
Reduction of Decimals.....99

LECTURE VII.

Reduction of Weights, Measures, &c.....104
Addition of Compound Numbers.129
Multiplication..do.....do....132
Subtraction of..do.....do....134
Division of.....do.....do....136

LECTURE VIII.

Fractions Simplified.....138
Preparatory Questions.....140
Decimal Fractions.....143
Ratio of Fractional parts of a $.143
Useful Theorems in Fractions..144
Addition of Fractions.....147
Reduction of mixed or Compound Fractions.....148

Page.

Multiplication of Fractions....149
Subtraction of Fractions.....151
Division of Fractions.....153
Ratio of mixed Numbers.....154
Rule of Three in Fractions....155
Inverse Proportion by Fractions 156

LECTURE IX.

Direct Proportion.....157
Contracted Operations.....161
Inverse Proportion.....162
Compound Proportion.....163
Practical Arithmetic.....164
How to Mark Goods.....168
Custom House Calculations.....170
Tare and Tret.....173
Interest.....176
Method of Calculation in Banking Institutions.....178
Partial Payments on Bills at interest.....183
Miscellaneous Questions.....186
Insurance.....188
Marine Insurance.....188
Insurance on Real Estate.....188
Merchandise, &c.....189
Commission.....190
Discount.....191
Buying and Selling Stocks.....192
Barter.....193
Loss and Gain.....195
Exercise Questions.....199
Company Accounts.....200
Apportioning the effects of a Bankrupt.....201
Equation of Payments.....203
General Averages.....205
How to Average personal accounts.....207

M577035

Page.
Exchange.....209
Value of Gold Coins.....210
European Coins reduced to dollars and cents.....211
Mercantile Forms.....212
Sterling money reduced to dollars and cents.....221
Exchange above or below par..223
Negotiating Bills of Exchange.225
Circular Exchange.....226
Involution.....227
Extraction of the Square Root..228
To find the distance that an object may be seen at sea.....230
Extraction of the Cube Root...231
Well known principles assumed.233
Single Position.....235
Double Position.....236
Annuities.....238
Annuities at Compound Interest.239
Annuities in Reversion.....243
Perpetuities.....244

Page.
Discount by Compound Interest.245
Alligation.....246
Arithmetical Progression.....247
Geometrical Progression.....249
Duodecimals.....251
Mensuration important to Ship Builders.....252
To find Ship's Tonnage.....254
Board or Lumber Measure.....254
To Measure Scantling or Joist..255
Papering Rooms.....256
Carpenter's work.....256
Bricklayer's work.....257
Mensuration of Superfices and Solids.....258
Guaging.....263
Abstract of mechanics.....264
Mechanical powers.....267
On the Steam Engine.....272
General Theorems.....274
Questions for Exercise.....275

RECOMMENDATION.

The following is from the pen of Richard Cotter, Esq., formerly Professor of Mathematics and Natural Philosophy, in Baltimore College.

I have carefully examined Mr. M'Nevins' manuscript of the "UNITED STATES CALCULATOR," and am happy to assert, that the various rules contained therein have been beautifully arranged, and that it contains a variety of matter useful to the student and accountant, not to be found in any Arithmetical work extant. Under these circumstances, I have no hesitation in saying, that it is a work of superior merit.

R. COTTER.

DEDICATION.

To the worthy instructers in the United States:—

GENTLEMEN—Permit me to present to your consideration the following system of Arithmetic which is intended as another auxiliary to your pupils in the useful department of mercantile education. The plan of teaching exhibited in the following sheets being theory and practice combined, is different from all which have hitherto been published, and can be communicated by the principal to his pupil with that success which experience alone, will sufficiently demonstrate. The work is in dollars and cents, adapted to all sorts of business, and nothing appears obscure or mysterious, but all is made plain and easy.

I remain gentlemen, yours respectfully,

JOHN M'NEVIN.

BALTO. Jan. 1, 1841.

PREFACE.

When I entered upon the arduous task of simplifying a science of so much importance to the civilized world as Arithmetic, in every department of society, I was aware, that it would be attended with no inconsiderable labour, deep thinking and research: How far I have succeeded to prove this work an auxiliary, to increase the general stock of knowledge already disseminated, (for the youth of our country) a generous, enlightened, and impartial public will determine. At all events, it is the result of much experience, during twenty-five years teaching, and it is the opinion of learned gentlemen to whom the manuscript has been submitted for inspection, that if it be introduced into our schools and academies, it will be found to answer admirably, the END, for which it is intended, and, thereby prove to be a guide and a text book to the merchant, mechanic and man of business, and, a standard system of Arithmetic in the United States.

Some persons may think, that books of a similar nature have been sufficiently multiplied; but in answer to this, I may with propriety state, that the only correct evidence of a spirit of improvement in our seminaries of learning, and that education is in a high state of prosperity, is, the continually increasing demand for new school books which exists in every section of our country. I ask then, what friend of our numerous, and flourishing literary institutions can, for a moment wish to check the progress of improvement in our books and systems of instruction;—Indeed

it may with perfect safety be mentioned, that when instructers become wholly content with the elements and principles which have LONG been in use the progress of improvement is entirely at an end.

It is a matter of fact that dispatch in business in every department, is no small accomplishment. Therefore, to prepare pupils in the shortest period of time possible for business, and to afford assistance to teachers in the arduous and important work of education were the motives which led me to this production. Let me ask, why is it that boys of common capacity can learn any kind of mechanical business in three or four years, when, in fact, double the time has been devoted to the attainment of only a *superficial* knowledge of the rudiments of Arithmetic? How is this, I pause for a reply? In the systems generally used in the schools of our country, the questions in the different rules are "*mixed*" up with examples of Pounds, Shillings and Pence. Certainly this is wrong!! Because the very idea of *pounds*, *shillings* and *pence* is foreign to American students. Not only this, but the derangement of the rules, and the *old-fashioned* protracted methods of calculation which are adopted in these old Arithmetics, retard the progress of pupils and cause a dislike to their studies. This is a fact, which cannot be controverted. And, that two-thirds of the time usually *spent* in acquiring a knowledge of this useful ART may be saved, by means of a proper system of Arithmetic and a correct mode of instruction is also, a fact well known to every judicious instructer. In no department of mathematical science have I been more deeply interested than in this, and hence, the reason, (as before mentioned) that I have laboured assiduously in preparing and collating a work, to suit the wants of the American people.

In relation to foreign currencies, *sterling money*, and the currencies of other countries of EUROPE are treated of in EXCHANGE: this is precisely as it should be: because, we

know that young persons are not lovers of *long* or *hard* lessons and that they CAN LEARN with pleasure, when neither their memories are overcharged nor their understandings put on the *rack*, from the use of tedious and ambiguous examples, which are interspersed throughout systems of Arithmetic *said* to be designed for American schools. In consequence of this, let us try to remedy this evil: In the first place, is it not well known, that the United States have political and civil institutions of their own, and can these be upheld unless, our children are taught to understand them by books and other means of instruction perfectly suited to the genius and constitution of the country? Again, a proper system of Arithmetic in dollars and cents, is much wanted, in order that pupils may receive from their teachers ample satisfaction for their TIME and MONEY. For it is manifest, that ALL, that concerns our public happiness, our union and peace within ourselves, ALL which tends to develope our resources, improve and perpetuate our institutions, ALL which may gives us wealth, strength, and glory among nations, depends on a general and systematic course of instruction in our schools. How ready, short, easy and familiar this work may be for actual performance, will appear by inspection.

In conclusion, I cheerfully tender my most grateful acknowledgments to RICHARD COTTER, ESQ. PROFESSOR OF MATHEMATICS AND NATURAL PHILOSOPHY OF THIS CITY, who has ably and faithfully acquitted himself in its EXAMINATION, and also, for the useful hints he has kindly suggested to me, which I conceive to be of utile importance to the work.

With these remarks, and an ardent wish, that the "UNITED STATES CALCULATOR; or, ARITHMETIC SIMPLIFIED" may be useful to the youth of my country, I cheerfully submit it to the patronage of an enlightened and independent public.

JOHN M'NEVIN.

BALTO. Jan. 1, 1841.

UNITED STATES CALCULATOR;

OR,

ARITHMETIC SIMPLIFIED.

ARITHMETIC is that part of the Mathematics which teaches the ART of computation by numbers. All operations in Arithmetic are performed by means of the following figures:

Cipher	One	Two	Three	Four	Five	Six	Seven	Eight	Nine
0	1	2	3	4	5	6	7	8	9.

There are five principal rules in Arithmetic, viz: Notation and Numeration may be included in 1, Addition 2, Multiplication 3, Subtraction 4, Division 5. The proper definitions are inserted at the head of each rule.

The Arabic numbers, are represented by the characters above written.

The Roman letters also express number without limitation, but are chiefly used to mark dates, and the chapters, and sections of books.

The following letters are used to express number.

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

NUMERATION TABLE.

Hundreds of Trillions.	Tens of trillions.	☞Trillions.☜	Hundreds of thousands of billions.	Tens of thousands of billions.	Thousands of billions.	Hundreds of billions.	Tens of billions.	☞Billions.☜	Hundreds of thousands of millions.	Tens of thousands of millions.	Thousands of millions.	Hundreds of millions.	Tens of millions.	☞Millions.☜	Hundreds of thousands.	Tens of thousands.	Thousands.	Hundreds.	Tens.	Units.
4	5	6	7	8	9	9	8	7	6	5	4	3	2	1	2	3	4	3	2	1

Any number of figures whatever, may be accurately numerated by dividing them into periods of six figures each as in the above table, repeating Millions of Millions for billions, millions of millions of millions for trillions, &c. as often as the figures 2, 3, 4, &c. placed underneath in the table designates.

ARCHIMEDES said "give me a place to stand and I shall remove the earth." The probable weight which Archimedes would have to raise is expressed by the following number of pounds, viz:

Trillions, Billions, Millions, lbs.

400,000,000,000,000,000,000,000

How many? Ans. 400,000 trillions of lbs.

LECTURE I.

PRELIMINARY REMARKS.

To use the language of "Lacroix," *the idea of Number is the latest and most difficult to form;* because, before the mind can arrive at such an abstract conception, it must be familiar with that process of classification, by which we successively remount from individuals to species, from species to genera and from genera to orders. The savage is lost in his attempts at numeration, and significantly expresses his inability to proceed by HOLDING UP HIS EXPANDED FINGERS or pointing to the hairs of his head.

Nature has furnished the great and universal standard for computation in the fingers of the hand. All nations have accordingly reckoned by FIVES, and some barbarous tribes have scarcely advanced any further. After the fingers of one hand had been counted once, it was a second, and perhaps, a distant step, to proceed to those of the other. The primitive words expressing numbers, did not probably exceed *five*. They counted by "*tally*" on wood, as the returns of our elections are made out on paper, thus 𝍸 for five, making four marks in a straight line, and drawing a diagonal line for the fifth.

To denote, *six*, *seven*, *eight* and *nine*, the North American Indians repeat the five, with the successive addition of one, two, three, and four. Could we trace the descent and affinity of the abbreviated terms denoting the numbers from five to ten, it seems highly probable, that we should discover a similar process to have taken place in the formation of the most refined languages. The ten

digits of both hands being reckoned up, it then became necessary to repeat the operation. Such is the foundation of our DECIMAL SCALE OF ARITHMETIC. Language still betrays, by its structure, the original mode of proceeding. To express the numbers beyond ten, the Laplanders combine an *ordinal* with a *cardinal* digit, thus eleven, twelve, &c. they denominate ONE TEN and *one*, one ten and *two*, &c.; and in like manner, they call twenty-one, twenty-two, &c. TWO TENS and one, TWO TENS and two, &c. which they express by holding up both hands twice for twenty, three times for THIRTY, four times for forty, and so on. When they want to express forty-three, or forty-four, it will be FOUR TENS and *three*, FOUR TENS and four, &c. Our term *eleven* is supposed to be derived from the Saxon *ein* or one and *liben* to remain, and to signify, *one leave or set aside ten*. *Twelve* is of the like derivation, and means *two laying aside ten*. The same idea is suggested by our termination *ty* in the words *twenty*, *thirty*, &c. This syllable altogether distinct from *ten*, is derived from *Ziehen* to draw, and the meaning of *twenty* is strictly speaking *two drawings*, that is, the hands have been twice closed and the fingers counted over. After *ten* was firmly established as the standard of numeration, it seemed, the most easy and consistent to proceed, by the same repeated composition. Both hands being closed, ten times would carry the reckoning up to *a hundred*. This word, originally *hund*, is of uncertain derivation; but the word *thousand* which occurs at the next stage of the progress, or the hundred added ten times, is clearly traced out, being only a contraction of *duis hund* or twice hundred, that is, the repetition or collection of hundreds.

NUMERATION.

DEFINITION.—The art of numbering, is that, which teaches the notation, and just method of reading numbers.

NUMBER, presents itself, in two different forms; FIRST,

as a WHOLE, without distinction of parts, as, when we speak of distance; as, the length of a line, from one extremity to the other.

SECONDLY, number is designated as a collection of things or parts, and one of these things or parts is called a unit, hence, the units are expressed, by *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine*. The COLLECTIONS of TEN UNITS or tens by 10, 20, 30, 40, 50, 60, 70, 80. The COLLECTIONS of 10 TENS or hundreds are expressed from names borrowed from the units, thus we say one hundred, two hundred, nine hundred, nine thousand, ten thousand, twenty thousand, one hundred thousand, two hundred thousand, &c. The collections of ten hundred thousand or of thousands of thousands, take the name of *millions*, and are distinguished like the collection of thousands.

1. *Write in figures*—Three hundred and eighty-six.

	hundreds.	tens.	units.
No. 1.	3	8	6

2. *Write in figures*—Nine thousand three hundred and seventy-three.

	thousands.	hundreds.	tens.	units.
No. 2.	9	3	7	3

3. *Write in figures*—Forty-three thousand four hundred and eighteen.

	tens of thousands.	thousands.	hundreds.	tens.	units.
No. 3.	4	3	4	1	8

4. One hundred and thirty-four thousand six hundred and thirty.

	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.
No. 4.	1	3	4	6	3	0

5. One million three hundred and eighty-six thousand four hundred and twenty-six.

	millions.	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.
No. 5.	1	3	8	6	4	2	6

6. Nineteen millions one hundred and seventy-four thousand eight hundred and seven.

	millions.	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.
No. 6.	19	1	7	4	8	0	7

7. Six hundred and fifty millions four thousand five hundred and five.

	millions.	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.
No. 7.	650	0	0	4	5	0	5

8. Six thousand five hundred and thirteen millions two hundred and eighty thousand three hundred and fourteen.

	millions.	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.
No. 8.	6513	2	8	0	3	1	4

When a number is written in figures, in enunciating or expressing it in language, it is necessary, to substitute for each of the figures, the word which it represents.

The following example will illustrate this:—

	tens of billions.	billions.	hundreds of thousands of millions.	tens of thousands of millions.	thousands of millions.	hundreds of millions.	tens of millions.	millions.	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.
	2	4	8	9	7	3	2	1	5	8	0	3	4	6

It is evident that the formation of numbers by the successive union of units, is independent of the units, as appears from the above table, by means of which, we are enabled to compound and decompound numbers, which is called CALCULATION. We shall now explain the principal rules, for the calculation of numbers, without regard to the nature of their units, and proceed to *Addition*.

LECTURE II.

Let us now, review this lecture, and enquire, what does the conjunction AND mean, when we say 4 *and* 5 make 9?

A. Addition.

Q. What then is addition?

A. The collecting or putting together of numbers.

Illustration.—6 *and* 8 are 14, which is called the sum.

Q. What is the sum called?

A. The amount or total aggregate.

Q. What difference is there between the SUM and the AMOUNT?

A. The *sum* means one particular sum, as 6 dollars, 8 dollars, or 10 dollars. And the amount is the sum total.

These have another meaning when we speak of interest. See Interest, lecture 10th.

ON ADDITION.

(Sign +)

It is a well known principle in *Mental Philosophy*, that the *mind* derives all its *primary ideas* from the immediate perception of the senses.

A knowledge of this fact shows the importance and utility of employing *sensible* or tangible objects, in order to assist the *juvenile capacity* in comprehending the nature and combinations of *abstract numbers*.

Nor should this be considered as a modern innovation, for we find, that the ancients frequently, had recourse to similar methods, when they attempted to teach the principles of Geometry. The Syrians, for instance, and most of the contiguous nations, looked to the Egyptians as teachers in most of the sciences. And it evidently appears, that their *instructions* and *communications* were imparted, chiefly, through the *medium* of *hieroglyphics*,

diagrams, and other figurative representations, adapted to their capacities and circumstances.

There is one important quality of the mind which deserves to be particularly noticed; for instance, when we speak of DOLLARS, we mean numerical increase, that is to say, ADDITION. Because, to add, is to increase from an unit.

Ten thousand of these American coins may be cast in the same die, and to all appearance may be precisely alike, neither does the mind conceive them under any idea of variety, but mere increase of number; from this principle, it may be laid down philosophically, that no individual can make great advances in intellectual improvement, beyond the immediate perception of *sensible objects*, such as pertain, to the five corporeal senses, *seeing*, *hearing*, *feeling*, *tasting*, and *smelling*.

This is a doctrine, to which our general train of reasoning will refer: for it is certain, there is no possibility of transmiting our ideas, but by reference, to sensible objects.

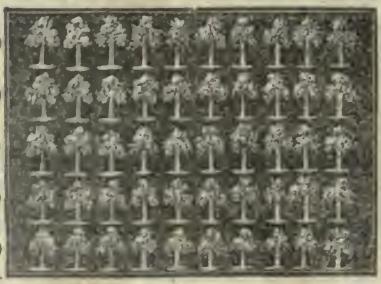
The science of Arithmetic in its present structure, can be reduced to mathematical precision, its elements can be analyzed, its combinations discovered, and a basis laid for gradual improvement, according to its general and consistent laws, by *Demonstrations*, and *Illustrations*.

Therefore, as the cultivation of the mind depends, chiefly on a correct habit of *thinking*, the teacher should, by all means convey his ideas in a plain and familiar manner, and in the commencement, have recourse to *visible objects*.

For instance, the diagram here laid down is a plan of an orchard, there are ten rows of apple trees in it, and five trees in each row, the question is, how many trees in the orchard? Ans. 50.

OPERATION.

1st line	5	
2nd row	5	10
3rd do.	5	
4th do.	5	10
5th do.	5	
6th do.	5	10
7th do.	5	
8th do.	5	10
9th do.	5	
10th do.	5	10



50 Ans.

Again, we next introduce, the system of addition by means of the CIRCULAR DIGITS. The teacher may commence with A in the class to count, in the following manner.

2	4	6	8	10	12	14	16	18	20	22	24
2	2	2	2	2	2	2	2	2	2	2	2
4	6	8	10	12	14	16	18	20	22	24	26
26	28	30	32	34	36	38	40	42	44	46	48
2	2	2	2	2	2	2	2	2	2	2	2
28	30	32	34	36	38	40	42	44	46	48	50

or differently, according to the numbers on the *Circular Digits* above mentioned.

Then, let the class commence with 3, 3+3=6.

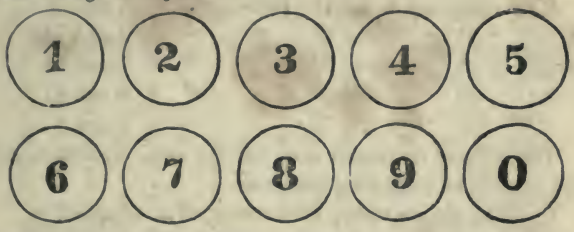
6	9	12	15	18	21	24	27, &c. up to 51.
3	3	3	3	3	3	3	3
9	12	15	18	21	24	27	30

Then again, with 4, 4 and 4 are 8,

8	12	16	20	24	28	32	36, &c. up to 52.
4	4	4	4	4	4	4	4
12	16	20	24	28	32	36	40

By this exercise, in small additions, pupils can readily answer in mental operations.

When the addition class is called up, each scholar should be furnished with a card, numbered agreeably to the lessons in the *Addition Table.* 1st card numbered the same as lesson first; 2d card numbered the same as lesson second, and so on. Hence, it is evident, that when, the subject of addition, is commenced, the pupils become highly delighted with this instructive and amusing process; because their ideas become interchanged, between one percipient thing and another, which, no doubt, emanate, from the use of these tangible objects.



The astonishing number, of changes, capable of being produced, by the various combinations of these elementary materials, afford, so many instances, of the ingenuity of man.

The nine figures, with a cipher 0. give all possible powers of numbers.

The figure A, B, C, D, is called a magic square.

8	1	6
3	5	7
4	9	2

The learner will find by adding the figures laterally, diagonally, and vertically, that the sum is 15.

It appears by inspection, that the central figure 5 is a *Magic number*, and the favourite figure in nature, for all the machinery on earth is reducible to the following 5 distinctive forms, viz: the *Wedge*, *Lever*, *Pulley*, *Wheel*, and the *Screw*.

Again, we have *five* fingers and *five* toes on each hand and foot, and we have, also, five senses, as before mentioned:—We find, that the ancients counted by *tally* on wood, the same as the returns of our elections are made out on paper by FIVES, and, the North American Indians count, by holding up their hands, so, that, nature has furnished, the great and universal standard, of addition by 5.

ADDITION is the first step to Mathematical knowledge, and is founded, on the first axiom, in Euclid, viz: the the whole is equal to its parts.

MENTAL EXERCISES.

How many are 5 and 4? How many are 8 and 5?
" " " 6 and 8? " " " 9 and 4?
" " " 6 and 3? " " " 7 and 5?
" " " 7 and 4? " " " 8 and 9?
" " " 9 and 7? " " " 9 and 9?

The learned instructer may use his own discretion, on the subject of proposing questions.

Q. Why do we carry, 1 for every 10 in addition?

A. The reason is manifest, from numeration, we find, that numbers increase in a ten fold ratio.

Example.—Gave $26 for a cow, and $9 for a load of hay, what did I give for both? Ans. $35 00.

	tens.	units.	
Operation,	2	6	Here 9 and 6 added together make
		9	15, which is 5 above 10.
Ans.	3	5	

I set down 5, in the units place, and carry 1 to 2 which make 3, which I set down in the ten's place and find the sum is 35.

DEFINITION.—To add, is to collect together, to make greater.

ADDITION TABLE.

First Lesson.

2 and 1 are 3
2 and 2 are 4
2 and 3 are 5

Second Lesson.

2 and 4 are 6
2 " 5 " 7
2 " 6 " 8
2 " 7 " 9

Third Lesson.

2 and 8 are 10
2 " 9 " 11
2 " 10 " 12
2 " 11 " 13
2 " 12 " 14

Fourth Lesson.

5 and 1 are 6
5 " 2 " 7
5 " 3 " 8
5 " 4 " 9
5 " 5 " 10
5 " 6 " 11

Fifth Lesson.

5 and 7 are 12
5 " 8 " 13
5 " 9 " 14
5 " 10 " 15
5 " 11 " 16
5 " 12 " 17

Sixth Lesson.

3 and 1 are 4
3 " 2 " 5
3 " 3 " 6
3 " 4 " 7
3 " 5 " 8
3 " 6 " 9
3 " 7 " 10

Seventh Lesson.

3 and 8 are 11
3 " 9 " 12
3 " 10 " 13
3 " 11 " 14
3 " 12 " 15

Eighth Lesson.

6 and 1 are 7
6 " 2 " 8
6 " 3 " 9
6 " 4 " 10
6 " 5 " 11
6 " 6 " 12

Ninth Lesson.

6 and 7 are 13
6 " 8 " 14
6 " 9 " 15
6 " 10 " 16
6 " 11 " 17
6 " 12 " 18

Tenth Lesson.

4 and 1 are 5
4 " 2 " 6
4 " 3 " 7
4 " 4 " 8
4 " 5 " 9
4 " 6 " 10
4 " 7 " 11

Eleventh Lesson.

4 and 8 are 12
4 " 9 " 13
4 " 10 " 14
4 " 11 " 15
4 " 12 " 16

Twelfth Lesson.

7 and 1 are 8
7 " 2 " 9
7 " 3 " 10
7 " 4 " 11
7 " 5 " 12
7 " 6 " 13

Thirteenth Lesson.

7 and 7 are 14
7 " 8 " 15
7 " 9 " 16
7 " 10 " 17
7 " 11 " 18
7 " 12 " 19

Fourteenth Lesson.

8 and 1 are 9
8 " 2 " 10
8 " 3 " 11
8 " 4 " 12
8 " 5 " 13
8 " 6 " 14

Fifteenth Lesson.

8 and 7 are 15
8 " 8 " 16
8 " 9 " 17
8 " 10 " 18
8 " 11 " 19
8 " 12 " 20

Sixteenth Lesson.

9 and 1 are 10
9 " 2 " 11
9 " 3 " 12
9 " 4 " 13
9 " 5 " 14
9 " 6 " 15

Seventeenth Lesson.

9 and 7 are 16
9 " 8 " 17
9 " 9 " 18
9 " 10 " 19
9 " 11 " 20
9 " 12 " 21

Eighteenth Lesson.

10 and 1 are 11
10 " 2 " 12
10 " 3 " 13
10 " 4 " 14
10 " 5 " 15
10 " 6 " 16

Nineteenth Lesson.

10 and 7 are 17
10 " 8 " 18
10 " 9 " 19
10 " 10 " 20
10 " 11 " 21
10 " 12 " 22

Twentieth Lesson.

11 and 1 are 12
11 " 2 " 13
11 " 3 " 14
11 " 4 " 15
11 " 5 " 16
11 " 6 " 17

Twenty-first Lesson.

11 and 7 are 18
11 " 8 " 19
11 " 9 " 20
11 " 10 " 21
11 " 11 " 22
11 " 12 " 23

Twenty-second Lesson.

12 and 1 are 13
12 " 2 " 14
12 " 3 " 15
12 " 4 " 16
12 " 5 " 17
12 " 6 " 18

Twenty-third Lesson.

12 and 7 are 19
12 " 8 " 20
12 " 9 " 21
12 " 10 " 22
12 " 11 " 23
12 " 12 " 24

Twenty-fourth Lesson.

13 and 1 are 14
13 " 2 " 15
13 " 3 " 16
13 " 4 " 17
13 " 5 " 18
13 " 6 " 19

Twenty-fifth Lesson.

13 and 7 are 20
13 " 8 " 21
13 " 9 " 22
13 " 10 " 23
13 " 11 " 24
13 " 12 " 25

DIVERTING EXERCISES IN ADDITION,

calculated to instruct and amuse the learner.

1	2	4	8	16	32
3	3	5	9	17	33
5	6	6	10	18	34
7	7	7	11	19	35
9	10	12	12	20	36
11	11	13	13	21	37
13	14	14	14	22	38
15	15	15	15	23	39
17	18	20	24	24	40
19	19	21	25	25	41
21	22	22	26	26	42
23	23	23	27	27	43
25	26	28	28	28	44
27	27	29	29	29	45
29	30	30	30	30	46
31	31	31	31	31	47
33	34	36	40	48	48
35	35	37	41	49	49
37	38	38	42	50	50
39	39	39	43	51	51
41	42	44	44	52	52
43	43	45	45	53	53
45	46	46	46	54	54
47	47	47	47	55	55
49	50	52	56	56	56
51	51	53	57	57	57
55	55	55	59	59	59
57	58	60	60	60	60
59	59	61	61	61	61
61	62	62	62	62	62
63	63	63	63	63	63

Elucidation of the foregoing Table.

On the foregoing page you see, that 1, 2, 4, 8, 16 and 32 are in the 1st horizontal row and no where else; but 1+2=3, shews that 3 is to be found in the 1st and 2nd columns, because it requires 1 and 2 to make 3. If it requires three or more of the top numbers to make one of the numbers in the table, as 1+2+4+8=15, the number 15 is to be found in all the columns in which 1, 2, &c. are found at top; and so it is with all the other numbers, which are generated in the same way. The horizontal rank of numbers, 1, 2, 4, 8, &c. are in geometrical ratio, and the perpendicular rank on the left in arithmetical ratio; both together *the abscisae and ordinates of the Logarithmic curve.* The rank of geometricals at the top viz: 1, 2, 4, &c., afford such combinations as make up all other numbers, in whatever column any number stands, the sum of the geometricals which stand over such a number is equal to it, and the reason, why one can guess any number thought on, is, the sum of the numbers which stand over it, in the first horizontal row, makes it, and no other exactly. Suppose for example, I fix upon the number 50, and ask you to tell, which number I thought on, you would ask me in which of the columns, and how many does it stand in? I would say in the second and two last columns, you would then put 2+16+32=50, and tell me to a certainty; because 50 is no where else, but in the second and two last columns. Whatever number you guess upon, if only, in the first, second, third, fourth, fifth, or sixth columns, the numbers must be 1, 2, 4, 8, 16 or 32, because each of them are found but once. But where it takes two or more of the figures in the horizontal row of geometricals, to make the one you think on, the number is to be found in all the columns, exactly under the geometricals, which compose it, and no where else, hence it is easy to understand the *diverting exercises*, if another column was added its head number would be 64, from

which a table could be generated from 1 to 127, because 63+64=127.

MENTAL EXERCISES.

1. How many are 5 and 4? 6 and 8? 6 and 3?

2. How many are 9 and 6? 8 and 9? 12 and 8?

Remark, in relation to the utility of proposing questions to be answered mentally in addition, the plan is very good, but in order to be expert in answering, the pupil should be required to recite the Addition Table. Mental exercises answer very well for small children, and for such, I would recommend "*Colburn's First Lessons.*"

1. Bought an orange for 3 cents, and nuts for 6 cents. How much did all cost?

2. Richard has 10 cents and Robert has 4 cents. How many have both?

3. You gave 12 cents for a knife, 4 cents for an inkstand and 1 cent for a slate pencil. How much will all of them come to?

4. If you have 4 nuts in one hand and 4 in the other. How many in both?

5. Thomas has 35 marbles in one pocket and 21 in the other. How many in both?

The teacher can propose similar questions to suit the capacity of the pupils.

DOLLARS AND CENTS.

The addition of dollars and cents is the same as the addition of whole numbers, with the exception of pointing out 2 figures, beginning at the units place for cents.

dolls. cts.	dolls. cts.	dolls. cts.
Add 371.25	Add 9863.37	Add 16.96
648.75	4962.74	44.92
934.63	3248.64	84.73
$1954.63		

dolls. cts.	dolls. cts.	dolls. cts.
Add 1349.16	Add 64669.74	Add 4.99
4469.74	44242.68	6.66
9324.68	93646.95	9.87
6466.67	44668.67	4.63
9464.63		

APPLICATION.

1. A gentleman gave $17.50 cents for a coat, $12 for pantaloons, $5.50 for a hat; what did they all come to?

Ans. $35.

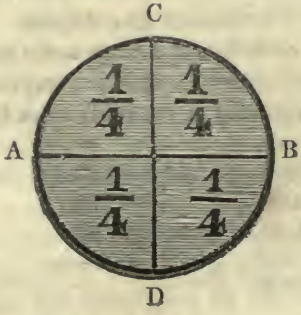
2. A merchant sent out his clerk to collect money; he collected $50 from one person, from a second $14.94, from a third $53.35, and fourth $31.43 cts.; how many dollars did he collect in all? Ans. $149.72 cts.

1. How many half cents in a cent? Ans. 2.
2. How many quarter cents in a cent? Ans. 4.
3. How many quarter cents in half a cent? Ans. 2.

ILLUSTRATION.

half or 2 quarters.

1 quarter.



dolls. cts.	dolls. cts.	dolls. cts.
Add 376.18½	Add 44.37½	Add 16.42½
244.36½	22.12½	17.96½
320.55		

dolls. cts.	dolls. cts.
Add 67942.37½	Add 6474.62½
42464.22¼	4248.68¼
49644.12½	4149.37¾

dolls. cts.	dolls. cts.
Add 6426846.37½	Add 33642.62½
4247494.18¾	94696.47¼
6424692.62½	92469.62¾

dolls. cts.	dolls. cts.
Add 64946.93¾	Add 924867.62½
44924.31¼	846493.68¾
94246.62½	924696.37½
84647.37½	448693.31¼

PROMISCUOUS EXAMPLES.

1. I owe a merchant $7.00 for sugar, $5 for coffee, $17 for flour, $3 for tea, and $2 for salt. What is the whole sum? Ans. 34 dollars.

2. A merchant bought 4 pieces domestic muslin, as follows; No. 1 contained 28 yards, for $2.80; No. 2 contained 29 yards for $3.19 cents; No. 3 contained 28¼ yards, cost $3.13½; No. 4, 27½ yards, cost $3.02½ cents. How many yards in the whole, and what did they all come to?

	yds.	dolls. cts.
No. 1.	28	2.80
2.	29	3.19
3.	28¼	3.13½
4.	27½	3.02½

A Butcher bought of one man 25 head of cattle for $450.87½, of another 15 head for $200.75, and of another 9 head for $77.62½. How many did he buy and what did the whole come to?

Ans. 59 head, whole cost $729.25.

ADDITION, OR AGREEABLY TO A MERCANTILE PHRASE, "FOOTING" OF DOLLARS AND CENTS.

A owes to one creditor 569 dollars, to another 3961, to another 581, to another 6116, to another 469, to another 506, to another 69381, and, to another 1261; what does he owe them all? Ans. $82871.00

The mode of footing here introduced, has been adopted in the Bank of the United States some years ago, and, of course, has been used by the best Book-keepers in the Union, on account of the plan, being not only simple, but ingenious.

OPERATION.

Place units	596	3	1	1st column.
under units,	3961	4	7	2nd "
tens under	581	3	8	3rd "
tens, & hun-	6116	2	2	4th "
dreds under	469	–	8	5th "
hundreds, &c.	506			
&c. &c.	69381			82871 as before.
	1261			
	82871			

ILLUSTRATION.

Place the addition of every column of figures to be added, under each other. Then when set down, reject the ten's place in every line except the LAST line.

EXAMPLE.

3	1
4	7
3	8
2	2
–	8

The ten's place being cut off, or rejected, in every line

except the last, there remains 82871. The lower figure 8 being set down first, 2 next, 8 next, 7 next, and 1 next; which written, will appear as above, 82871.

	dolls. cts.
Add	4168.34
	7816.56
	8423.46
	9814.34
	6813.49
	$37036.19

As before, add up every column in succession, which place under each other, and reject or cut off the ten's place, and you have a correct addition in one line.

EXAMPLE 2.

Addition of the	1st	column is	2	9	
" " "	2nd	"	2	1	
" " "	3rd	"	2	6	
" " "	4th	"	1	3	
" " "	5th	"	3	0	
" " "	6th	"		37	
				$37036.19	

ANOTHER METHOD.

By adding up, every line, simply without carrying to the units place, but "*skipping*" one figure back, every time, until you get through. A sufficient proof in itself, that units must be placed under units, tens under tens, hundreds under hundreds, and THOUSANDS under thousands, &c., or cents under cents, and dollars under dollars, in the same order, thus:

Addition of first line,		29
" "	2nd	19
" "	3rd,	24
" "	4th,	11
" "	5th,	29
" "	6th,	34
		$37036.19

3. A man owes Ezekiel Dorsey $6374.18 cts.; to B. M'Donough $4397.65 cts.; to John E. Stansbury $9365 94 cents, and to the Merchants' Bank $7986.44 cents; how much does he owe all? Ans. $28124.21.

4. A man bought a barrel of flour for $5.75, a barrel of molasses for $29.00, and a barrel of rum for $36.00; how much did he pay for all the articles. Ans. $70.75.

5. From the creation of the world to the flood, was 1656 years; from thence to the building of Solomon's Temple, 1340 years; thence to the birth of our Saviour, 1008 years. In what year of the world was the birth of Christ? Answer. Anno Mundi, (in the year of the world) 4004 years.

EXTENSIVE ADDITIONS,

dolls. cts.	EXAMPLE.		
Add 148697.69	Addition of the 1st line is	5	0
448644.44	" 2nd "	5	4
934444.78	" 3rd "	4	5
945464.64	" 4th "	6	3
948492.69	" 5th "	4	4
947498.75	" 6th "	5	7
947698.63	" 7th "	3	7
456493.78	"		
	" 8th "		57
$5777435.40			

In rejecting the ten's place in every line except the last, the remainder is 57774354, which is set down for the answer.

dolls. cts.
Add 64668.16
94648.74
94844.65
47486.94
94646.64
94648.63
Add 94666846.63
46494646.74
94646468.64
94646462.63

dolls. cts.
Add 64967.63
94648.62
94738.64
94646.68
94484.62
Add 96486.63
46486.64
93287.64
64968.69
69696.87
69486.63

Q. How are we to place figures in addition?

A. Units, under units, tens under tens, hundreds under hundreds, thousands under thousands, &c.

dolls. cts.	dolls. cts.
Add 6469.86	Add 6469.64
4649,87	9469.77
4698.75	9469.78
9468.69	9464.63
6469.68	9448.97
9464.99	9864.63
6469.68	
9464.77	dolls. cts.
	Add 64698.69
dolls. cts.	46949.64
Add 6498746.68	94674.69
6469696.75	92324.64
9446946.68	94646.69
9467469.75	94698.84
9646975.44	92465.75

LECTURE III.

ON MULTIPLICATION.

(Sign ×.)

Q. What is multiplication?

A. Multiplication is a concise way of performing extensive ADDITIONS; because, to multiply, is to increase one number by another, as often, as there are units, in that number, by which the one is increased.

Q. How many parts has it?

A. THREE; the Multiplier, Multiplicand and the Rectangle or Product.

Q. Which number is the multiplier?

A. The number you multiply by.

The reader will find, that the MULTIPLICATION TABLE, in this work, is different from the common form, and, that, the system, now, being introduced, for teaching the elements of figures, in multiplication, leads to great improvement in mental researches. Inasmuch as, it unites THEORY with PRACTICE; for, although, multiplication in itself, is simple, its minor variations are endless. Because, the relative value of figures, connected with the workings, of *thought*, adapt themselves, to every imaginable form, and *run into each other*, by such nice gradations, as are only obvious, to the keenest observations of philosophy; and yet all this amazing system of intellectual improvement, depends, on the combinations of nine figures, with a cipher, called, the nine digits.

MULTIPLICATION TABLE.

2 co	2	3	4	5	6	7	8	9
1	2	2	2	2	2	2	2	2
2	4	6	8	10	12	14	16	18
3 times	2	3	4	5	6	7	8	9
1	3	3	3	3	3	3	3	3
3	6	9	12	15	18	21	24	27
4 times	2	3	4	5	6	7	8	9
1	4	4	4	4	4	4	4	4
4	8	12	16	20	24	28	32	36
5 times	2	3	4	5	6	7	8	9
1	5	5	5	5	5	5	5	5
5	10	15	20	25	30	35	40	45
6 times	2	3	4	5	6	7	8	9
1	6	6	6	6	6	6	6	6
6	12	18	24	30	36	42	48	54
7 times	2	3	4	5	6	7	8	9
1	7	7	7	7	7	7	7	7
7	14	21	28	35	42	49	56	63
8 times	2	3	4	5	6	7	8	9
1	8	8	8	8	8	8	8	8
8	16	24	32	40	48	56	64	72
9 times	2	3	4	5	6	7	8	9
1	9	9	9	9	9	9	9	9
9	18	27	36	45	54	63	72	81

Q. What is a multiple?

A. It is a term in multiplication when one number, is contained in another several times; as, 15 is the multiple of 3. Containing it 5 times.

Q. What is the multiple of 24?

A. 4, 6, 8, &c.

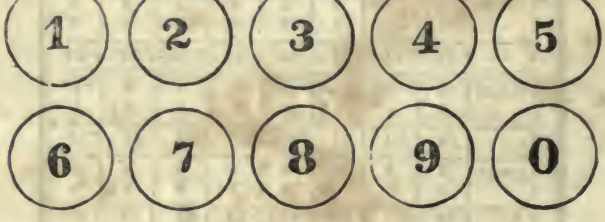
Q. How can the product of two numbers be expressed in multiplication?

A. As $3 \times 5 = 15$ product.

Q. In this case, which is the multiplicand?
A. 3.
Q. Which is the multiplier?
A. 5.
Q. Can either be a multiplier?
A. Yes? because $3 \times 5 = 5 \times 3 = 15$ as above.

Mode of reciting the Table.—The learner should be taught to recite thus: beginning with 2 ce—2 ce 1 are 2, 1, or once, 2 are 2; 2 ce 2 are 4, and repeat, 2 ce 2 are 4; 2 ce 3 are 6, *vice versa*, 3 times 2 are 6; 2 ce 4 are 8, 4 times 2 are 8; 2 ce 5 are 10, 5 times 2 are 10; 2 ce 6 are 12, 6 times 2 are 12; 2 ce 7 are 14, 7 times 2 are 14; 2 ce 8 are 16, 8 times 2 are 16; 2 ce 9 are 18, 9 times 2 are 18.

Again, begin with 3 times, 3 times 2 are 6, 2 ce 3 are 6; three times 3 are 9. Repeat, 3 times 3 are 9, 3 times 4 are 12, 4 times three are 12, and so on, alternately.



SYSTEM OF TEACHING.

Let the teacher procure 8 sets of '*circular digits*,' as above represented, numbered agreeably to the form of the multiplication table.

	First lesson.					*Second lesson.*			
First set.	2	2	3	4	5	6	7	8	9
	1	2	2	2	2	2	2	2	2
	2	4	6	8	10	12	14	16	18

	Third lesson.					*Fourth lesson.*			
Second set.	3	2	3	4	5	6	7	8	9
	1	3	3	3	3	3	3	3	3
	3	6	9	12	15	18	21	24	27

The circles to be numbered on both sides as above: CARD No. 1, to shew lesson 1st on one side, and lesson 2d on the other. CARD No. 2, to represent lesson 3d on one side, and lesson 4th on the other, and so on. By this plan, children five or six years of age, can be taught to multiply and answer mentally in a few days.

CASE I.—*Point two places of figures for cents, beginning at the right hand, counting from right to left.*

1. Sold 16 lbs. of Sugar at 6 cts.
   14 " " " 7 "
   12 " " " 6 "
   10 " " " 8 "
   —— $3.46

2. Sold 8 lbs. of Sugar at 7 cts.
   6 " " " 8 "
   5 " " " 10 "
   3 " " " 8 "
   1 " " " 9 "
   Whole amount, —— $5.33

3. Sold 3 yds. at $3.00
   4 " " 5.00
   9 " " 6.00
   7 " " 9.00
   —— $146.00

4. Sold 15 lbs. at 8 cts.
   18 " " 10 "
   16 " " 11 "
   18 " " 12 "
   19 " " 15 "
   —— $9.77

5. A lady bought the following items, &c.

13 yds.	Calico	at	22 cts.
8 "	"	"	32 "
5 "	Domestic muslin	"	13 "
40 "	Muslin	"	17 "
			—— $12.87

6. Bo't 8 yds. Tape at 5 cts.
10 " " " 8 "
11 " " " 9 "
12 " " " 8 "
—— $3.15

7. Sold 10 lbs. of Coffee at 10 cts.
12 " " " 9 "
15 " " " 12 "
—— $3.88

8. Sold 16 yds. Muslin at 11 cts.
12 " " " 13 "
15 " " " 14 "
9 " " " 13 "
8 " " " 15 "
—— $7.79

9. A lady bought the following articles, viz:

8 yds.	Calico	at	31 cts.
12 "	Domestic muslin	"	15 "
6 "	Canton flannel	"	20 "
3 "	Ribband	"	15 "
9 "	Bombazine	"	44 "
			—— $9.89

10. A boy bought the following items, viz:

2	Slates	at	27 cts.
3	Copy Books	"	10 "
25	Quills	"	2 "
100	Slate pencils	for	20 "
			—— $1.54

11. Sold 16 yds.	at	9 cts.	
14 "	"	8 "	
17 "	"	9 "	
16 "	"	11 "	
17 "	"	7 "	
			$7.04
12. Sold 14 lbs.	at	9 cts.	
13 "	"	8 "	
12 "	"	7 "	
13 "	"	6 "	
14 "	"	9 "	
15 "	"	8 "	
			$6.38
13. Sold 13 lbs.	at	5 cts.	
16 "	"	4 "	
19 "	"	7 "	
18 "	"	8 "	
19 "	"	6 "	
24 "	"	8 "	
			$7.12
14. Sold 19 lbs.	at	15 cts.	
22 "	"	16 "	
23 "	"	17 "	
24 "	"	18 "	
32 "	"	19 "	
45 "	"	21 "	
			$30.13

15. Bought 10 pieces of muslin, each 29 yards, at 22 cents. What is the amount? Ans. $63.80.

16. Sold 5 bags of coffee each weighing 156 lbs. at 15 cents. Ans. $117.00

17. Bought 125 barrels of flour at $4.81 per barrel. Ans. $601.25

18. Sold 125 cords wood at $6 pr. cord. Ans. $750.00

19. Sold 144 bbls. apples at 75c. per lb. Ans. $108.00

20. Bought 162 bushels of oats at 44 cents. What is the amount? Ans. $71.28.

21. Sold 132 yards cassinett at 87 cents. What is the amount? $114.84

22. Sold 19 yds. at 37 cts.
22 " " 31 "
25 " " 42 "
28 " " 45 "
44 " " 65 "
—— $65.55

23. Sold to Charles B. Needles,
3 yds. green cloth at $5.25, per yd. $15.75
4 yds. Muslin at 35 cts. per yd. 1.40
$17.15

CASE II.

MENTAL OPERATIONS.

1. What will 3 oranges come to at 4 cents a piece? Ans. 12c.

2. What will 6 quarts of cherries come to at 5 cents per quart? Ans. 30c.

3. What will 6 pounds of sugar come at 9 cents per pound? Ans. 54c.

4. What will 8 yards of ribbon come to 8 cents per yard? Ans. 64c.

5. If a man travel 3 miles an hour, how many miles will he travel in 5 hours? Ans. 15

6. How many will he travel in 6 hours? Ans. 18

7. How many in 8 hours? Ans. 24

8. There are 12 inches in a foot, how many in 2 feet? Ans. 24

9. How many inches in 4 feet? Ans. 48

10. How many inches in 8 feet? Ans. 96

11. There are 3 feet in a yard.

12. How many feet in 7 yards? Ans. 21

13. How many feet in 9 yards? Ans. 27

14. How many feet in 12 yards? Ans. 36

15. There are 7 days in a week. How many days in 8 weeks? Ans. 56

16. How many days in 9 weeks? Ans. 63

17. A league is 3 miles, how many miles in 91 leagues? Ans. 273

18. What will 15 pounds of sugar come to at 10 cents? Ans. $1.50

The annexed diagram of an orchard is applicable in this case, to assist the learner in Multiplication.

There are 10 rows of apple trees in it, and 5 trees in each row. The question is, how many trees in the orchard? Ans. 50

Here there are 10 rows, and 5 trees in each row.

Multiply 10
By 5
—
Ans. 50 trees.
—

From the above diagram we find that INCREASE consists of *Addition* and *Multiplication.*

QUESTIONS FOR EXERCISE.

1. Calculate the amount of 126 lbs. butter at 15 cents per pound? $18.90

2. Calculate the amount of 19 cords of wood at $3.75 per cord? $71.25

3. 169 bbls. cider at $2.12 per barrel? $358.28

4. 63 lbs. tea at $.93 per pound? $58.59

5. 789 lbs. sole leather, at 19 cts. per lb? $149.91

6. 47 thousand shingles at $3.75 per M? $176.25

7. 169 boxes of oranges at $6.71 per box? $1133.99
8. 17 score penknives at 17 cts. each? $57.80
9. 96 yds. broad cloth at $5.75 per yard? $552.00
10. 421 bushels wheat at $1.35 per bushel? $568.35
11. Sold 6 bags of coffee per lb. as follows:

No. 1	160 lbs.	at	15 cts.	per	lb.	$24.00
2	130 "	"	15	"		19.50
3	140 "	"	14	"		19.60
4	120 "	"	13	"		15.60
5	148 "	"	12	"		17.76
6	139 "	"	11	"		15.29
						$111.75

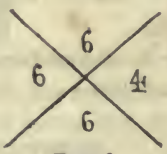
'Sums' in which there are Ciphers at the right hand of the Multiplier or Multiplicand.

RULE.—Cut off the ciphers and add them to the result when multiplied.

No. 1
123
2(0
2460

No. 2
64(00
4(00
2560000

No. 3
987(000
148(000
7896
3948
9878
146076000000



Proof.

Sums in which Ciphers are intermixed.

RULE.—Skip one place for every cipher.

No. 1
106
204
424
212
21624

No. 2
120013
408
978415
50208

No. 3
374957
2009
526228
400026

No. 4

No. 5

No. 6 40085 × 3024

No. 7 4240 × 3109

Sums in which Ciphers are intermixed with the figures of the Multiplier.

1. A merchant bought 106 hhds. of wine at $204 per hhd. How many dollars did he pay for all?

Multiplicand, - - 106
Multiplier, - - 204

424
212

21624 Product.

No. 21 20013 × 408

No. 3 347957 × 2009

No. 4 978415 × 50208

No. 5 526228 × 70016

No. 6 356984 × 400026

Sums in which the Multiplier is 10, 100, 1000, &c.

1. A merchant bought 76 barrels of flour at $10 per barrel. How many dollars did he pay for the whole?

Multiplicand, - - 76
Multiplier, - - 10

Product, 760

No. 2 43128 × 10

No. 3 8653 × 100

No. 4 213 × 1000

No. 5 29187 × 10000

No. 6 8674 × 100000

No. 7 435000 × 9350000

EXERCISE QUESTIONS.

1. If you pay $7 for a barrel of flour, how many dollars must you pay for 125 barrels? Ans. $875.00

2. If you pay 8 cents for one lb. of pork, how many cents must you pay for 375 pounds? Ans. $30.00

3. A merchant bought 225 lemons at 5 cents a piece, how many cents did he pay for the whole? Ans. $11.25

4. James has 125 nuts, and William has 5 times as many, how many nuts has William? Ans. 625

5. A merchant has 9 boxes of raisins, each containing 15 pounds, how many pounds do all the boxes contain? Ans. 135 lbs.

6. A farmer sold 428 pounds of cheese at 7 cents a pound, how many cents did he receive for the whole? Ans. $29.96

7. If you pay $7 for one yard of broad cloth, how many dollars must you pay for 6 yards? Ans. $42.00

8. If the wages of one man for a year be $132, what will the wages of 12 men be? Ans. $1584.00

9. There are 24 hours in a day, and 7 days in a week, how many hours in one week? Ans. 168

10. There are in a hhd. 63 gallons, how many gallons in 24 hogsheads? Ans. 1512

11. To the Senate of the United States each of the 26 States, sends two members, how many members are there in the Senate of the United States? Ans. 52

12. If a steamboat run 15 miles in one hour, how many miles can it run in 24 hours! Ans. 360

13. In 20 years how many days, allowing a year to be 365 days? Ans. 7300

14. What will 168 reams of medium paper come to at $4.25 per ream? Ans. $714.00

15. Bought 120 boxes of oranges at $5.35 per box. Ans. $642.00

16. 148 yds. broad cloth at $3.75 pr. yd. Ans. $555.00

17. 95 yds. cassinett at 62½ cts. pr. yd. Ans. $59.37½

18. 144 yds. of broad cloth at $3.95 pr. yd. $568.80

19. James Young, Bought of John Wilson,

12 lbs. loaf sugar,	at	17 cts.
10 " brown do.	"	9 "
12 " coffee,	"	15 "

Received payment, —— $4.74

QUESTIONS FOR THE SLATE.

1. What will 14 lbs. sugar come to at 13 cents per lb?
2. 16 yds. at 17 " per yd.
3. 19 " " 15 " "
4. 64 " " 55 " "
5. 83 " " 86 " "

—— $113.97

Sold to John R. Kemp,

6. 74 gals. of wine at 72 cts. per gal.
7. 98 " " 91 " "
8. 43 square feet of boards at 45 cts. per foot.
9. 54 bushs. of corn at 62 " per bush.
10. 85 " " " 87 " "
11. 97 gallons of brandy at 95 " per gal.

Amount, —— $361.39

Bought of David Simmons,

12. 23 yds. " 25 cts. per yard?
13. 28 " " 29 " "
14. 36 " " 37 " "
15. 48 " " 43 " "
16. 43 " " 48 " "

—— $68.47

Bought of Ross Patterson,

17. 46 lbs. of coffee at 19 cts. per lb.
18. 49 yds. " 46 " per yd.
19. 54 " " 59 " "
20. 59 " " 54 " "
21. 99 " " 92 " "

Amount, —— $186.08

22. 109 yds. at $1.08 cts. per yard?
23. 117 " " 1.15 " "
24. 119 bushels of wheat at 1.18 " per bush.
25. 34 " of corn at 37 " "
26. 56 " " 54 " "
27. 125 " of wheat at 1.25 " "

—— $591.76

28. What will 28 yds. muslin come to at 22 cts. pr. yd?
29. 25 yds. at 28 " "
30. 17 " " 23 " "
31. 37 " " 33 " "
32. 33 " " 36 " "
33. 47 " " 46 " "
34. 48 " " 47 " "

—— $85.34

Sold to George Harwood,

35. 77 lbs. of tea at 80 cts. per lb?
36. 94 " " 95 " "
37. 97 gallons of wine " 93 " per gal.
38. 88 " " " 87 " "
39. 95 " " " 94 " "
40. 79 bushels of corn " 76 " pr. bush.

Amount, —— $552.35

CALCULATION OF BILLS.

Sold John H. Sharp,

3 yds.	broad cloth	at	$5 87 cts. per yard.
5 "	brown holland	"	33 " "
10 "	padding	"	62 " "
18 "	red flannel	"	43 " "

—— $33.20

Sold Oliver J. Griffin,

460 bales	cotton net,	138,700 lbs.	at 24 cts. per lb.
210 "	"	63,200 lbs.	at 22 " "
250 "	"	75,400 lbs.	at 23 " "

—— $64534.00

Sold Thomas R. Steele,

3 sugar loaves, wt. 51 lbs.	at	17 cts. per lb.
19 lbs. of rice	"	8 " "
15 lbs. of pepper	"	23 " "
33 lbs. of cloves	"	11 " "

—— $17.27

Sold William Hebden,

18 yds.	calico	at	37 cts.
34 "	muslin	"	15 "
63 "	linen	"	73 "
56 "	flannel	"	44 "
39 "	cassimere	"	87 "

—— $116.32

Bought of Cornelius Ryan,

84 cords	oak wood	at	$3.75
38 "	hickory	"	5.62
58 "	yellow pine	"	2.62

—— $680.52

Bought of Messrs. Wheelright, Turner, & Mudge,

64 reams	of	medium	paper	at	$2.75
96 "	"	"	"	"	3.25
84 "	"	"	"	"	3.50
66 "	"	"	"	"	4.25

—— $1062.50

Bought the following bill of goods, viz:

16 yds.	domestic	muslin	at	15 cts.	
12 "	"	"	"	18 "	
19 "	"	"	"	17 "	
25 "	"	"	"	37 "	

—— $17.04

Sold Samuel James,

12 yds. superfine cloth at $6.25 per yard.
3 pieces linen cont'ing 37 yds. at 60c. "

—— $97.20

Bought of Henry Harwood,

3 yds.	blue cloth	at	$4.25 cts.
14 "	muslin	"	11 "
8 skeins	silk	"	6 "

—— $14.77

Sold James Young,

2 pieces Irish linen cont'ing 46 yds. at 75 cts. pr. yd.

3 "	Canton flannel	"	60	"	13	"	"
1 "	silk	"	36	"	63	"	"

— $64.98

Sold Isaac Shirk,

9 bbls. superfine flour	at	$4.75	cts.	per bbl.
8 " " " "		5.25	"	"
84 " " " "		5.33	"	"
42 half bbls. " " "		5.28	"	"
		——	$643.35	

Sold G. W. Goddard,

1 bag Laguayra coffee,	wt. 160 lbs.	at	10	cts. pr lb.
1 " Java "	wt. 144 lbs.	at	11	" "
1 " Rio "	wt. 135 lbs.	at	11	" "
1 " St. Domingo "	wt. 120 lbs.	at	10	" "
			—	$58.69

Bought of J. S. Barry,

16 yds. cassinett	at	75	cts.	per yard.
84 " bombazine	"	44	"	"
81 " flannel	"	56	"	"
36 " muslin	"	14	"	"
		—	$99.36	

13. Sold 49 green hides at $1.95 cents each, what did they come to? Ans. $95.55

BILLS.

14. Philadelphia, January 3, 1840.

Mr. Jacob Miller, Bought of P. Mercer,

12 yds. superfine cloth	at	$5.30	cts.	per yard.
2 pieces linen cont'ing 37 yds.	at	80	"	"
		——	$93.20	

Baltimore, January 1, 1840.

Mr. Joseph Ryan, Bought of P. Mercer,

6 silk cravats	at	$1.50	cts.	each.
1 doz. pair kid gloves	"	75	"	"
10 yds. black velvet	"	2.60	"	per yard.
3 red Merino shawls	"	12.50	"	each.
		———	$81.50	

MERCANTILE CALCULATIONS.

Baltimore, June 4, 1840.

Mr. Adam Cook, Bought of Job Lewis,

2 pieces Irish linen, cont'ing	46 yds. at	$1.20	pr. yd.
3 " Canton flannel "	95 yds. at	60	"
1 " silk "	30 yds. at	50	"

—— $127.20

Baltimore, January 3, 1840.

Mr. Isaac Williams, Bought of Thomas Ferrel,

17 yds. flannel	at	45 cts.	per yard.
13 lbs. tea	"	98 "	per lb.
47 " cheese	"	12 "	"

— $26.03

Baltimore, June 5, 1840.

Mr. Jonas Bailey, Bought of P. Mercer,

19 yds. superfine cloth	at	$6.40	per yard.
20 " linen	"	1.12	"
30 " muslin	"	15	"
2 black silk cravats	"	1 40	each.

—— $151.30

Baltimore, June 6, 1840.

Mr. Job Lewis, Bought of P. Mercer,

37 yds. cassimere	at	$1.60,	
40 " muslin	"	17,	
A lot of trimmings.	Amount	per bill	$27.43
A lot of ribbons.	"	"	6.00
			$99.43
19 yds. lace	at	$2.27	per yard.
14 " ribbon	"	18	"
24 " "	"	25	"
13 fans	"	13	each.

—— $53.34

BILLS.

Baltimore, June 6, 1840.

Mr. John Abbott, Bought of S. Brewer,

17 pieces Irish linen, cont'ing 216 yds. at $1.10 pr. yd.
15 yds. book muslin at 56 "
—— $246.00

Baltimore, June 7, 1840.

Mr. Thomas Clark, Bought of Samuel Brown,

12 yds. muslin at 38 cts. per yard.
10 " calico " 62 " "
6 " drab cloth " $4.75 " "
——— $39.26

Baltimore, June 7, 1840.

David Lusby, Bought of P. Fenby,

25 gallons of rum at $1.25 per gal.
16 lbs. candles " 15 per lb.
11 " soap " 14 "
—— $35.19

Baltimore, July 7, 1840.

Mr. Samuel A. W. Campbell, Bought of E. Dorsey,

3 M (thousand) quills at $7.50
4 doz. spelling books " 2.25
1 ream of paper " 3.00
—— $34.50

Baltimore, June 7, 1840.

Mr. George Mitchell, Bought of John Andrews,

12 yds. muslin at 20 cts. per yard.
10 " calico " 28 " "
3 " cassimere " 87 " "
— $7.81

Baltimore, June 7, 1840.

Mr. Wm. Kemp, Bought of G. P. Knotts,

20 pieces of hanging paper at 50 cts.
8 " superior " 1.00 "
—— $18.00

Baltimore, June 8, 1840.

Mr. Richard Bruff, Bought of E. Dorsey,

13 lbs. loaf sugar	at	20 cts.	per lb.
2 doz. cups and saucers	"	$2.75	per doz.
26 lbs. coffee	"	15 cts.	per lb.
36 " brown sugar	"	10 "	"
4 " Imperial tea	"	1.25 "	"
5 " mustard	"	15 "	"
2 " green tea	"	87 "	"
			$23.09

Baltimore, June 8, 1840.

Mr. Nicholas King, Bought of E. Dorsey,

2 doz. knives and forks	at	$2.50	per doz.
3 pair sad irons	"	1.25	
½ doz. waiters	"	2.00	
1 castor	"	3.00	
			$13.75

Baltimore, June 8, 1840.

Mr. James Chandler, Bought of Sam'l Brown,

1 piece of long lawn, 20 yds.	at	$1.12	per yd.
6 yds. linen cambric	"	4.50	"
			$49.40

Baltimore, June 10, 1840.

Mr. E. C. Johnson, Jr. Bought of Sam'l Brown,

2 pieces American nankeen, 11 yds.	at	31 cts.	pr. yd.
2 pair cotton hose	at	50 "	pr. pr.
2 " silk "	"	1.00 "	"
2 " gloves	"	1.00 "	"
1 piece cambric muslin, 12 yds.	at	1.00 "	per yd.
			$20.41

Baltimore, June 11, 1840.

Mr. Andrew Simpson, Bought of Robt. Broom,

3 bbls. superfine flour	at	$6.75	per bbl.
3 bushels of corn	"	85	per bush.
8 " oats	"	33	"
			$25.44

Baltimore, June 10, 1840.

Mr. Wm. James Donohue, Bo't of Thos. Daugherty,

1	piece Irish Linen, 25 yds.	at	$1.12
1	" Bandanna hdkfs.	"	10.50
1	" shirting muslin, 35 yds.	"	37
6	" calico, each 29 yds.	"	29
4	" Russia sheeting, cont. 120 yds.		45

—— $155.91

Baltimore, June 10, 1840.

Mr. Wm. Patridge, Bought of Thos. Greaves,

28 yds. of broad cloth	at	$5.60 per yd.
15 bbls. of flour	"	6.70 per bbl.
90 gals. of molasses	"	46 per gal.
14 lbs of coffee	"	15 per lb.

—— $300.80

Baltimore, June 10, 1840.

Mr. John Cantwell, Bo't of Francis Jordan,

7 lbs. coffee	at	14 cts. per lb.
9 " tea	"	38 " "
50 " fish	"	5 " "
7 gallons of wine	"	1.25 " per gal.
1 bbl. oil	"	23.82 "

——— $39.47

Baltimore, June 11, 1840.

Mr. Thomas Bruff, Bought of George Conway,

16 lbs. coffee	at	25 cts. per lb.
8 " sugar	"	11 " "
15 " hyson tea	"	42 " "
24 " loaf sugar	"	17 " "

—— $15.26

Baltimore, June 11, 1840.

Mr. Wm. T. Beeks, Bought of James Mooney,

14 yds. tow cloth	at	8 cts. per yard.
40 " brown linen	"	25 " "
4 pieces of nankeen	"	1.87 " "
9 yds. striped jean	"	20 " "

—— $20.40

Canton, July 11, 1840.

Mr. David Johnson, Bought of Sam'l Smith,

13 lbs.	tea	at	98 cts.	per lb.
16 "	coffee	"	15 "	"
36 "	sugar	"	13 "	"
47 "	cheese	"	9 "	"
12 "	pepper	"	19 "	"
7 "	ginger	"	17 "	"
13 "	chocolate	"	61 "	"

—— $35.45

FARMERS' ACCOUNTS.

Dr. William Wallace, Jr. To Norman Nash,

To 11 bbls.	cider	at	$3.00
" 14 lbs.	butter	"	33
" 52 "	dried beef	"	10
" 83 "	cheese	"	9
" 4	bushels of apples	"	25
" 3	firkins of butter, each 115lbs.	at	17

—— $109.94

LECTURE IV.

ON SUBTRACTION.

(Sign —)

It is a fact well known to Mathematicians, that the principles of ARITHMETIC and ALGEBRA, admit of INCREASE, DECREASE, and EQUALITY. *Increase* consists of ADDITION and MULTIPLICATION; *decrease*, of SUBTRACTION and DIVISION. *Equality*, (being the result or answer agreeably to the *data* of the question, under different names,) is represented by a statement or equation, reduced by *increase* or *decrease*, to the lowest term or answer. Hence, to facilitate increase or decrease, nothing remains but to simplify expressions or statements. After having learned to compose a number by the addition and multiplication of several others, agreeably to the laws of *increase*, the first question that presents itself is, how to take one number from another that is greater; or, in other words, to separate the greater number into two parts, one of which shall be the given number.

This is called the doctrine of *decrease*, and may be illustrated thus: Suppose we wish to take $3 from $10, by so doing we separate $10 into two parts, one of which shall be $3 the given number; then we begin with 10, the greater number in question, and descend as many places from 10 as there are units in the lesser number, and we shall come to the number required, which is $7. Hence, we find that 7 is the *excess* of 10 above 3, or we might

say, that 7 is the *difference* or *remainder* between 10 and 3. Consequently, the words *excess*, *difference*, and *remainder*, are synonymous, each answering to the separation of $10 into the parts, $3 and $7, which is always designated by the name SUBTRACTION.

DEFINITION—TO SUBTRACT IS TO MAKE LESS.

(Sign —)

Q. What is the upper line denoting the greater number called?

A. The MINUEND.

Q. What is the lower line denoting the lesser number called?

A. The SUBTRAHEND. The *difference* of both is called the remainder.

RULE.

1. Place the lesser number under the greater, so that units may appear under units, tens under tens, and hundreds under hundreds, &c. and draw a line underneath.

2. Begin at the right hand, and take each figure in the lower line from the figure above it, and set down the remainder.

3. If the lower figure is greater than that above it, add 10 to the upper number, from which number, so increased, take the lower, and set down the remainder, carrying ONE to the next lower number, with which proceed as before, and so on, till the whole is finished.

PROOF.

Add the remainder to the lesser number, and if the sum be equal to the greater, the work is right.

SUBTRACTION TABLE.

First Lesson.

From	1	2	3	4	5	6	7	8	9	10
Take	1	1	1	1	1	1	1	1	1	1
Rem.	0	1	2	3	4	5	6	7	8	9

Second Lesson.

From	2	3	4	5	6	7	8	9	10	11
Take	2	2	2	2	2	2	2	2	2	2
Rem.	0	1	2	3	4	5	6	7	8	9

Third Lesson.

From	2	4	5	6	7	8	9	10	11	12
Take	2	3	3	3	3	3	3	3	3	3
Rem.	0	1	2	3	4	5	6	7	8	9

Fourth Lesson.

From	4	5	6	7	8	9	10	11	12	13
Take	4	4	4	4	4	4	4	4	4	4
Rem.	0	1	2	3	4	5	6	7	8	9

Fifth Lesson.

From	5	6	7	8	9	10	11	12	13	14
Take	4	4	4	4	4	4	4	4	4	4
Rem.	1	2	3	4	5	6	7	8	9	10

Sixth Lesson.

From	6	7	8	9	10	11	12	13	14	15
Take	5	5	5	5	5	5	5	5	5	5
Rem.	1	2	3	4	5	6	7	8	9	10

Seventh Lesson.

From	7	8	9	10	11	12	13	14	15	16
Take	6	6	6	6	6	6	6	6	6	6
Rem.	1	2	3	4	5	6	7	8	9	10

Eighth Lesson.

From	8	9	10	11	12	13	14	15	16	17
Take	7	7	7	7	7	7	7	7	7	7
Rem.	1	2	3	4	5	6	7	8	9	10

Ninth Lesson.

From	9	10	11	12	13	14	15	16	17	18
Take	8	8	8	8	8	8	8	8	8	8
Rem.	1	2	3	4	5	6	7	8	9	10

Tenth Lesson.

From	10	11	12	13	14	15	16	17	18	19
Take	9	9	9	9	9	9	9	9	9	9
Rem.	1	2	3	4	5	6	7	8	9	10

1. Henry has 25 marbles and James 9, how many has Henry more than James? Ans. 16.

2. Thomas bought 10 oranges and gave 3 to William, how many had he left? Ans. 7.

3. Bought a book for 35 cents, and sold it for 25 cents, how much did I lose? Ans. 10c.

4. Bought a penknife for 75 cents and sold it for $1, how much did I gain? Ans. 25c.

5. If you give 25 cents for a knife and 20 cents for a book, how many cents did the knife cost more than the book? Ans. 5c.

6. The Arabian method of Notation was discovered in England about the year 1150, how long is it since to this period. Ans. 690.

7. Lucas De Burgo, discovered the method of book-keeping, by Double Entry, at Venice in 1495, how long is it since to this present year, 1840? Ans. 345.

8. If my son FERDINAND AUGUSTINE, had lived, he would be 3 years old on the 28th day of August, 1839, what time would he be 21 years old?

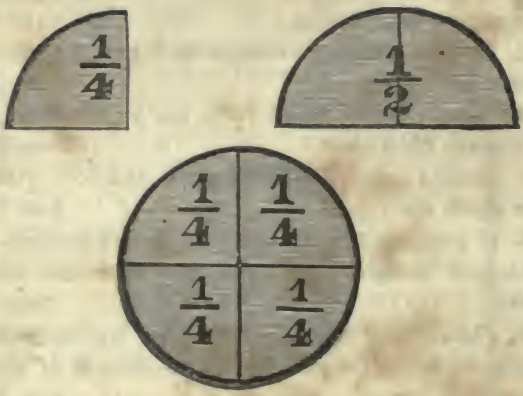
Ans. August 28, 1857.

9. From $100 borrowed take $72 paid. 'Twas a lady lent it, what's due to the maid? Ans. $28.00.

10. From 90 take 30, from 40 take 10. Subtract 6 from 100, and what remains then? Ans. 184.

SUBTRACTION OF DOLLARS AND CENTS.

1.	From	$166.94	From	$9474.94
	Take	149.68	Take	6496.69
	Rem'r.		Rem'r.	
2.	From	$9476.97	From	$6944.69
	Take	8469.69	Take	4649.79
	Rem'r.		Rem'r.	



The above cuts represent fractional parts of a cent and may serve to illustrate *Subtraction*, as well as *Addition*, inasmuch, as the latter can be proved by subtraction.

EXAMPLE.

From	$978 73¼	Here ¼ taken from ½	
Take	644.37¼	leaves ¼.	
	$334.36¼		
Proof. Add	$644.37¼		
	334.36¼		
Total,	$978.73¾	as above.	

This sum denoting the upper line, is called the *minuend*, or greater number.

1. From $100 take ¼ of a cent? Ans. $99.99¾
2. From $200 take 12½ cents? Ans. $199.87½
3. From $99.93¾ cents take $63.18½? Ans. $36.75¼
4. From $200 take 18¾ cents? Ans. $199.81¼
5. From $130.37½ cts. take $31.18¾? Ans. $99.18¾
6. From $199.11¾ cts take 93¾ cts.? Ans. $198.18

Example.—From \$137.98*c*.7*m*. take \$13.94*c*.8*m*.

Ans. \$124.03.9

Operation,

From	\$137.98.7
Take	13.94.8
Ans.	\$124.03.9

In the above example, composed of dollars, cents, and mills, the operation is performed the same as whole numbers, except the pointing.

Q. How is the pointing performed?

A. By commencing at the units, and pointing off one place for mills, and two for cents. The reason is obvious, because there are 1000 mills in a dollar, which is equivalent to 100 cents.

1. From \$193.98.5 take \$37.63.9. Ans. \$156.34.6
2. From \$349.87.5 take \$5.62.9. Ans. \$344.24.6
3. From \$869.93.9 take \$8.67.9. Ans. \$861.26
4. From \$1000.08.9 take \$9.68.9. Ans. \$990.40

QUESTIONS FOR THE BLACK BOARD.

1. Subtract 37½ cents from \$10.00? Ans. \$9.62½
2. Subtract 43¾ cents from \$100.00?
3. Subtract 18¾ cents from \$5.00?
4. Subtract \$1.06¼ from \$18.00?
5. Subtract \$3.37½ from \$6.93¾?
6. Subtract \$3.31¼ from \$10.00?
7. Subtract \$1.12½ from \$74.00?
8. Subtract \$1.56¼ from \$100.00?

1.	From	139	Take	64	Remainder	75
2.	"	86	"	49	"	37
3.	"	128	"	99	"	29
4.	"	402	"	315	"	87
5.	"	616	"	144	"	472
6.	"	156	"	129	"	27
7.	"	124	"	56	"	68
8.	"	96	"	37	"	59
9.	"	214	"	177	"	37

QUESTIONS FOR EXERCISE.

ADDITION AND SUBTRACTION.

1. Sent a servant to market with $2 to buy provisions, who bought meat for 60 cents, butter 32 cents, cheese 28 cents, and fowls 30 cents, how much change must he return? Ans. 50c.

2. Gave 25 cents for paper, 12½ cents for quills, and 37½ cents for an arithmetic, what change must I have out of a dollar note? Ans. 25c.

3. Borrowed $25.00 and paid $17.00, how much do I owe? Ans. $8.00

MULTIPLICATION AND SUBTRACTION.

1. Bought the following items, viz:

16 lbs.	sugar	at	9 cts.	per	lb.
10 "	cheese	"	14 "	"	
4 "	butter	"	25 "	"	
6 "	coffee	"	13 "	"	
			—— $4.62		

2. I handed the merchant a five dollar bill, how much is the '*change*?' Ans. 38c.

3. Bought

2 yds.	muslin	at	20 cts.	per	yard.
6 "	calico	"	25 "	"	
3 "	ribband	"	16 "	"	
3	cotton balls	"	6 "		
8 yds.	green silk	"	80 "	"	
			—— $8.96		

4. A merchant received a ten dollar note on the Marine Bank in payment of the above bill, how much is the '*change*?' Ans. $1.04

LECTURE V.

ON DIVISION.

(Sign ÷)

DIVISION consists of *Addition*, *Subtraction*, and *Multiplication*, in its operation, and in finding one of the factors of a given product, when the other is known; like SUBTRACTION, it is also, founded upon the principles of DECREASE, and can be illustrated thus: If it be required to ascertain the number of times, 24 contains 4. We need only subtract 4 from 24, as many times as it can be done; and since, after 6 subtractions nothing is left, we conclude 4 is contained in 24, 6 times; this manner of decomposing one number by another, in order to know how many times the last is contained in the first, is called DIVISION; because, it serves to divide, or portion out, a given number into equal parts of which the number or value is given. The number to be divided is called the *dividend*, the factor that is known, and by which we must divide, is called the *divisor*, the factor found by the division is called the *quotient*, and always shows how many times the divisor is contained in the dividend. Hence, it is evident, that the *divisor*, multiplied by the *quotient*, will reproduce the dividend.

DIVISION OF THE NINE DIGITS ILLUSTRATED.

2 into	4	5	6	7	8	9	10	11
Quot'nt	2	$2\frac{1}{2}$	3	$3\frac{1}{2}$	4	$4\frac{1}{2}$	5	$5\frac{1}{2}$
3 into	1	2	3	4	5	6	7	8
Quot'nt	$0\frac{1}{3}$	$0\frac{2}{3}$	1	$1\frac{1}{3}$	$1\frac{2}{3}$	2	$2\frac{1}{3}$	$2\frac{2}{3}$
4 into	1	2	3	4	5	6	7	8
Quot'nt	$0\frac{1}{4}$	$0\frac{1}{2}$	$0\frac{3}{4}$	1	$1\frac{1}{4}$	$1\frac{1}{2}$	$1\frac{3}{4}$	2
5 into	1	2	3	4	5	6	7	8
Quot'nt	$0\frac{1}{5}$	$0\frac{2}{5}$	$0\frac{3}{5}$	$0\frac{4}{5}$	1	$1\frac{1}{5}$	$1\frac{2}{5}$	$1\frac{3}{5}$
6 into	1	2	3	4	5	6	7	8
Quot'nt	$0\frac{1}{6}$	$0\frac{2}{6}$	$0\frac{3}{6}$	$0\frac{4}{6}$	$0\frac{5}{6}$	1	$1\frac{1}{6}$	$1\frac{2}{6}$
7 into	1	2	3	4	5	6	7	8
Quot'nt	$0\frac{1}{7}$	$0\frac{2}{7}$	$0\frac{3}{7}$	$0\frac{4}{7}$	$0\frac{5}{7}$	$0\frac{6}{7}$	1	$1\frac{1}{7}$
8 into	1	2	3	4	5	6	7	8
Quot'nt	$0\frac{1}{8}$	$0\frac{2}{8}$	$0\frac{3}{8}$	$0\frac{4}{8}$	$0\frac{5}{8}$	$0\frac{6}{8}$	$0\frac{7}{8}$	1
9 into	1	2	3	4	5	6	7	8
Quot'nt	$0\frac{1}{9}$	$0\frac{2}{9}$	$0\frac{3}{9}$	$0\frac{4}{9}$	$0\frac{5}{9}$	$0\frac{6}{9}$	$\frac{7}{9}$	$0\frac{8}{9}$
10 into	1	2	3	4	5	6	7	8
Quot'nt	$0\frac{1}{10}$	$0\frac{2}{10}$	$0\frac{3}{10}$	$0\frac{4}{10}$	$0\frac{5}{10}$	$0\frac{6}{10}$	$0\frac{7}{10}$	$0\frac{8}{10}$

☞ The learner should be taught to set the remainders in a fractional form, and to know that a remainder is so many parts of the divisor; thus divide 6 by 7, and it leaves a remainder of 6, the remainder is 6 parts of 7, and is written $\frac{6}{7}$.

Q. When you divide by 2, and have 1 of a remainder, what is the remainder called? Ans. $\frac{1}{2}$

Divide by 3, and have 1, of a remainder. Ans. $\frac{1}{3}$

Divide by 3, and have 2. Ans. $\frac{2}{3}$

Divide by 4, and have 1, and so on. Ans. $\frac{1}{4}$

Q. When we divide an apple into eight equal parts, what will each part be called?

A. $\frac{1}{8}$ into 9, into 10, into 11, into 12.

Q. What is the number given to divide by called?

A. The divisor or denominator of a fraction.

Q. What is the number to be divided by called?

A. The dividend or numerator of a fraction.

Q. What is that which is sometimes left after dividing, or when the operation is performed called?

A. The remainder.

Q. What is the number of times that the divisor is contained in the dividend called?

A. The quotient. The dividend is a multiple of either the divisor or quotient, because the quotient × by the divisor is equal to the dividend.

Q. If you have a remainder, must that remainder be less than the divisor?

A. It must, or else there is an error.

Q. If your dividend be dollars, what will your remainder be?

A. Dollars.

Q. If ounces, if cents, if pounds, if yards, &c? If years, if months, if weeks, if days, if hours, if minutes? What does the remainder shew?

A. So many fractional parts of the divisor, and it must be of the same name with the dividend.

Q. Divide $5 by 4, the quotient will be $1, and the remainder $1, what is that remainder called?

A. ¼ of a dollar. Divide $8 by 7, the quotient will be 1, and the remainder 1, which is ⅐ of a dollar, or of the divisor.

Q Can you '*go*' higher than 9 times in division?

A. You cannot.

Q. Why so?

A. Because we have but 9 figures with a cipher 0 (as before mentioned in addition,) in our numerical calculations, as 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. Consequently, the figure 9, is the highest number, which could possibly be placed in the quotient.

Q. How many parts in division?

A. Three, the *divisor*, *dividend*, and *quotient*.

Q. How is division proved?

A. It may be proved by casting out the 9's as observed in addition, or by multiplication; hence, multiplication and division mutually prove each other.

Q. In what way?

A. Because the rectangle or product of the divisor must evidently be equal to the *dividend*.

Q. Can multiplication be proved by division?

A. It can, by dividing the rectangle or product of the multiplier and multiplicand, by either as $16 \times 12 = 192$ product. Here, $\frac{192}{12} = 16$, or $\frac{192}{16} = 12$.

EXAMPLE.

Divisor.	Dividend.	Quotient.
2)	6	(3
	6	
	0	

Divisor.	Dividend.	Quotient.
3)	6	(2
	6	
	0	

Because 2 is contained in 6 (3 times, and 3 is contained in 6) 2ce, it is manifest that 3 times 2 are 6, and that 2 ce 3 are 6. Again:

Divisor.	Dividend.	Quotient.
4)	8	(2
	8	
	0	

Divisor.	Dividend.	Quotient.
2)	8	(4
	8	
	0	

Here 4 is the divisor, 8 the dividend, and 2 the quotient, which shews that 4 is contained in 8, (2 times in like manner 2 is contained in 8 four times. Therefore, by reference to the multiplication table, we find that 2 ce 4 are 8, and 4 times 2 are 8.

DIRECTIONS FOR TEACHERS.

As soon as the pupil can recite the division table thoroughly, agreeably to the plan here laid down, let him be exercised in mental operations. Again, the next step will be to find the factors of given numbers, thus: 6 is a factor of 24, how is the other found? Ans. from the table we know that $6 \times 4 = 24$, consequently, 4 is the other factor, and so on.

Divisor.	Dividend.	Quotient.	Proved.
Thus: 3)	into 9	(3 times, because 3 times	3 are 9
" 3)	into 12	(4 times, because 4 times	3 are 12
" 3)	into 15	(5 times, because 5 times	3 are 15
" 3)	into 18	(6 times, because 6 times	3 are 18
" 3)	into 21	(7 times, because 7 times	3 are 21
" 3)	into 24	(8 times, because 8 times	3 are 24
" 3)	into 27	(9 times, because 9 times	3 are 27
" 4)	into 12	(3 times, because 3 times	4 are 12
" 4)	into 16	(4 times, because 4 times	4 are 16
" 4)	into 20	(5 times, because 5 times	4 are 20
" 4)	into 24	(6 times, because 6 times	4 are 24
" 4)	into 28	(7 times, because 7 times	4 are 28
" 4)	into 32	(8 times, because 8 times	4 are 32
" 4)	into 36	(9 times, because 9 times	4 are 36
" 5)	into 5	(1, because 1 or once	5 are 5
" 5)	into 10	(2, because 2 or twice	5 are 10
" 5)	into 15	(3 times, because 3 times	5 are 15
" 5)	into 20	(4 times, because 4 times	5 are 20
" 5)	into 25	(5 times, because 5 times	5 are 25
" 5)	into 30	(6 times, because 6 times	5 are 30
" 5)	into 35	(7 times, because 7 times	5 are 35
" 5)	into 40	(8 times, because 8 times	5 are 40
" 5)	into 45	(9 times, because 9 times	5 are 45
" 6)	into 6	(1 or once, "	1 or once 6 are 6
" 6)	into 12	(2 or twice, "	2 or twice 6 are 12
" 6)	into 18	(3 times, because 3 times	6 are 18
" 6)	into 24	(4 times, because 4 times	6 are 24
" 6)	into 30	(5 times, because 5 times	6 are 30
" 6)	into 36	(6 times, because 6 times	6 are 36

DIRECTIONS FOR TEACHERS CONTINUED.

Divisor.	Dividend.	Quotient.	Proved.
Thus: 6)	into 42	(7 times,	because 7 times 6 are 42
" 6)	into 48	(8 times,	because 8 times 6 are 48
" 6)	into 54	(9 times,	because 9 times 6 are 54
" 7)	into 7	(1 or once,	" 1 or once 7 are 7
" 7)	into 14	(2 or twice,	" 2 or twice 7 are 14
" 7)	into 21	(3 times,	because 3 times 7 are 21
" 7)	into 28	(4 times,	because 4 times 7 are 28
" 7)	into 35	(5 times,	because 5 times 7 are 35
" 7)	into 42	(6 times,	because 6 times 7 are 42
" 7)	into 49	(7 times,	because 7 times 7 are 49
" 7)	into 56	(8 times,	because 8 times 7 are 56
" 7)	into 63	(9 times,	because 9 times 7 are 63
" 8)	into 8	(1 or once,	" 1 or once 8 are 8
" 8)	into 16	(2 or twice,	" 2 or 2 ce 8 are 16
" 8)	into 24	(3 times,	because 3 times 8 are 24
" 8)	into 32	(4 times,	because 4 times 8 are 32
" 8)	into 40	(5 times,	because 5 times 8 are 40
" 8)	into 48	(6 times,	because 6 times 8 are 48
" 8)	into 56	(7 times,	because 7 times 8 are 56
" 8)	into 64	(8 times,	because 8 times 8 are 64
" 8)	into 72	(9 times,	because 9 times 8 are 72
" 9)	into 9	(1 or once,	" 9 times 1 are 9
" 9)	into 18	(2 or twice,	" 9 times 2 are 18
" 9)	into 27	(3 times,	because 9 times 3 are 27
" 9)	into 36	(4 times,	because 9 times 4 are 36
" 9)	into 45	(5 times,	because 9 times 5 are 45
" 9)	into 54	(6 times,	because 9 times 6 are 54
" 9)	into 63	(7 times,	because 9 times 7 are 63
" 9)	into 72	(8 times,	because 9 times 8 are 72
" 9)	into 81	(9 times,	because 9 times 9 are 81

☞ Let young scholars be lectured regularly, agreeably to the above plan, in order to be perfect in mental operations.

SHORT DIVISION.

RULE.

1. Place the divisor to the left of the number you wish to divide.

2. Consider how many times the number by which you divide is contained in the first figure or figures of the number to be divided, and set down the result, noting if there be a remainder.

3. If there be no remainder, consider how often the divisor is contained in the next figure, but if there be a remainder, conceive it to be placed to the left of the next figure, into which divide as before and set down the result.

APPLICATION OF THE RULE.

Ex. 1.	Ex. 2.	Ex. 3.	Ex. 4.	Ex. 5.
2)482	2)648	3)963	4)484	2)236
241	324	321	121	118

Here in the 5th example, when we divide 2 into 2, the quotient is 1, which set down, but dividing 2 into 3 the next figure, the quotient is 1 and 1 over, which place before the following figure 6, and 16 is represented; therefore, 2 into 16 is contained 8 times. Hence, the quotient is 118 as above.

Ex. 6.	Ex. 7.	Ex. 8.	Ex. 9.
4)484	5)750	6)756976974	7)876942875
121	150		

Ex. 10.	Ex. 11.
8)96869875	9)867487549874

The teacher is requested to give the pupil as many questions of this kind as he may think proper.

PRACTICAL QUESTIONS.

1. Two boys, Dick and Harry, have 12 apples which they divide equally, how many will each have? Ans. 6

2. What is the quotient of 8736, divided by 8 and by 4? . Ans. 273

3. Thomas gave 72 cents for 8 quarts of cherries, what was the price per quart?

4. If four boys, John, James, William, and Robert, get 28 apples, how many will each have?

5. If Richard buys a slate for 6 cents, how many can he buy for 30 cents?

6. If a man travel 3 miles in an hour, how many hours will it take him to travel 36 miles?

7. At 6 cents a piece for oranges, how many can you buy for 48 cents?

MENTAL EXERCISES.

Q. How much is the half of $5.00.

A. The half of 5 is 2 and 1 over, which is one dollar, and the half of a dollar is 50 cents; therefore, the half of $5.00 must be two dollars and fifty cents.

Q. How much is the half of $7.00?

Q. How much is the half of $11.00?

Q. How much is the third of $1.00? Ans. 33⅓c.

Operation, 3)1.00

33⅓ cts.

☞ It is well known to the youth of our country that 100 cents is a dollar, and to the weakest capacity, that if an apple, a number or any thing be divided by 3, the quotient will be ⅓ of the dividend, therefore, $33\frac{1}{3} \times 3 = 100$. In the above case, it will be found that 3 would be a constant divisor, were ciphers to be annexed to the remainder *ad infinitum*, but by the principles of division when the divisors are constant, the quotients will vary as

the dividends vary, the quotients, therefore, resulting from finite quantities divided by 0 will vary as those quantities vary, and as finite quantities may be infinitely varied, an infinity of infinites is proven. Because it is well known to *Mathematicians*, that any finite quantity whatever, divided by 0 will give infinites for a quotient, Q. — E. — D.

Divide 2687676 by 2.	Ans. 1343838
Divide 8469674 by 3.	" 2823224⅔
Divide 967485 by 4.	" 241871¼
Divide 764756 by 5.	" 152951⅕
Divide 997349 by 6.	" 166224⅚
Divide 874966 by 7.	" 124995⅐
Divide 934365 by 8.	" 116796⅞
Divide 77634 by 9.	" 8626

DIVISION OF FEDERAL MONEY.

Divide $40.75 by 2.	Ans. $20.37½
Divide 39.31 by 3.	" 13.10
Divide 41.37 by 4.	" 10.34¼
Divide 48.76 by 5.	" 9.75⅕
Diviee 37.45 by 6.	" 6.24⅙
Divide 44.18 by 7.	" 6.31⅐
Divide 50.49 by 8.	" 6.31⅛
Divide 100.00 by 9.	" 11.11⅑
Divide 50.61 by 10.	" 5.06$\frac{1}{10}$
Divide 55.67 by 11.	" 5.06$\frac{1}{11}$
Divide 60.73 by 12.	" 5 06$\frac{1}{12}$

Q. How do you write down one-half?

A. ½—I write 1, draw a line under it, and place 2 below the line, what is the figure 1 called? Ans. the numerator.

Q. What is the figure 2 called?

A. The denominator.

EXERCISE QUESTIONS.

Divide	2	into 14	Ans. 7 times, and no remainder.
"	4	" 18	Remainder 2
"	6	" 26	" 2
"	8	" 46	" 6
"	9	" 37	" 1
"	10	" 44	" 4
"	11	" 23	" 1
"	11	" 68	" 2

CASE II.

When we divide by a composite number, say 12, 14, 16, 18, 21, 36, 43, 63, &c.

Divide 144 by 12—4 times 3 are 12, now 4(144

3)36

12 Ans.

Here we divide by 4 and by 3, because $4 \times 3 = 12$.

Example 2.—Suppose we want to divide 196 by 14; here $2 \times 7 = 14$. Hence, we divide 196 by 2 and by 7, and the result will be the same as if divided by 14.

Thus: 7)196

2)28

14 Answer.

EXERCISES FOR THE SLATE.

Divide	144	by	16
"	160	"	40
"	256	"	64
"	567	"	24
"	625	"	25
"	1728	"	144
"	2744	"	196

DIVISION AND SUBTRACTION.

1. A gentleman made a present of $224 to his three daughters, in the following manner: to the eldest he gave $80, and to the other two girls each, one-half, how much did each get?

A. The oldest $80, and each of the other girls $72.

2. A captain, mate, and 25 sailors, received a prize of $30.000, the captain got one-half, the mate one half what was left, and the remainder was to be divided equally between the sailors, how much did each seamen get?

A. Captain, $15,000, mate, $7,500, and each of the sailors, $300.

QUESTIONS IN WHICH THERE ARE CIPHERS AT THE RIGHT HAND OF THE DIVISOR.

EXAMPLE.

A gentlemen sold a farm for $4,540, at $10 per acre, how many acres did the farm contain? Ans. 454

RULE.—*Cut off as many ciphers from the divisor as there are ciphers in the dividend.*

Divisor Dividend. Quotient.

Thus: 1(0)454(0 (454 Ans.

1. A gentlemen had a number of men in his employment, to whom he paid $1,895.00, and each man received $250, how many men were there? Ans. 758

2. If 350 bushels of corn cost $217, what is it per bushel? Ans. 62c.

3. If 1000 gallons of molasses cost $430, what is it per gallon? Ans. 43c.

4. One hundred and forty-four men have to pay equal shares of a debt which amounts to $14400, how much must each man advance to make up the sum? Ans. $100.

QUESTIONS IN WHICH THE DIVISOR IS A COMPOSITE NUMBER.

Example.—A farmer sold 28 cows for $448, how much did he receive for each cow? Ans. $16

ILLUSTRATION.

It will be readily perceived that 28 is a *composite* number, and that its factors are 4 and 7, as appears from the multiplication table, which is also a *table of factors*. Here, 28 divided into factors will be equal to 7×4, and hence, we find that $448 divided by 7, will give 64 for a quotient, and that quotient, divided by 4 will give 16, exactly the same as if we divided $448 by 28.

Operation, 7)448 or 28)448(16
4)64 28
168
Answer, 16 168

So that from the mode of cancelling, it appears that 1 refers to 16; consequently, one cow cost $16.

1. A lady paid $17.15 cts. for a quantity of cambric at 49 cents a yard, how many yards did she buy?

The factors are 7 and 7, because $7 \times 7 = 49$.

Divisors.
7)1715
7)245
35 Answer.

2. A merchant bought a quantity of beef for which he paid $6,480, at $15 a barrel, how many barrels did he buy? Ans. 432.

Five and three are the common factors for 15, because $5 \times 3 = 15$, consequently, we divide by

5)6480
3)1296
432 dollars.

QUESTIONS FOR THE SLATE.

Divide 10464 by 24?	Ans. 436
" 16704 by 36?	" 464
" 33792 by 48?	" 704
" 62496 by 56?	" 1116
" 70119 by 63?	" 1113
" 80064 by 72?	" 1112
" 93660 by 84?	" 1115

MISCELLANEOUS EXAMPLES,

To exercise the learner in ADDITION, SUBTRACTION, MULTIPLICATION *and* DIVISION.

1. A farmer sold 487 bushels of wheat at $2 a bushel, what is the amount? Ans. $974.

2. How long will it take a man to travel 264 miles if he should travel 24 miles a day? Ans. 11.

3. In the field of battle there were 268 soldiers in rank, and 118 in file, what number of soldiers were there in the Army? Ans. 31624.

4. A gentleman had four bags of money, the first bag contained $3,475 the second $6,950, the third $934, and the fourth $467, how many dollars were there in the bags? Ans. $11,826.

5. A man bought 4875 bushels of wheat, and sold 1899 bushels, how many bushels had he left? Ans. 2976.

6. A merchant bought 198 barrels of flour at $5.00 a barrel, how many dollars did he pay for the whole? Ans. $990.

7. The sum of $2,000 was equally divided among 25 men, how many dollars did each man receive? Ans. 80.

8. A gentleman had 24 houses, and received 75 dollars rent, per annum, for each, how much did all his rents amount to? Ans. $1800.

9. A gentleman divided his farm containing 895 acres, equally among his five sons, how many acres did each receive for his portion? Ans. 179.

10. The earth moves round the sun at the rate of 69000 miles in an hour, how many miles does it travel in 24 hours? Ans. 2875.

11. A merchant bought a bill of goods to the amount of $545, and paid at the time of purchase $450, how many dollars remain unpaid? Ans. $95.00

12. How many acres are there in a piece of land 39 acres in length, and 20 in breadth? Ans. 780.

13. Suppose a man to earn $35 a month, how many months will it take him to earn $490. Ans. 14.

14. If $1,024 be divided equally among 64 men, how many dollars will one man receive? Ans. $16.00

15. In a certain school there are 36 scholars, among whom 540 quills are to be equally divided, how many will 1 scholar receive? Ans. 15.

16. If $40 pay for an acre of land, how many acres can be bought for $680? Ans. 17.

17. A farmer planted 2072 trees in 14 equal rows, how many did he plant in a row? Ans. 148.

18. If 35 yards of cloth cost $315, what would be the cost of one yard? Ans. $9.

19. If $774 be divided equally among 18 sailors, how many dollars will each sailor receive? Ans. $43.

20. If a man travel 48 miles a day, in how many days will he perform a journey of 3264 miles? Ans. 68.

21. If a man travel 24 miles a day, in how many days will he perform a journey of 816 miles? Ans. 34.

11. A drover received $1188 for 44 head of cattle, how much did he get for each? Ans. $27.

22. The wages of 36 men for 1 month, is $540, how much will each get? Ans. $15.

23. Suppose a regiment of 648 men have 9720 pounds of beef, how many lbs. are there for each man? Ans. 15.

24. If 35 barrels of flour cost $210, how much did 1 barrel cost? Ans. $6.

25. If a regiment of 512 men have 8192 pounds of beef, how many pounds for each man? Ans. 16.

Here we find $64 \times 8 = 512$; hence, as 64 is a composite number we get $8 \times 8 \times 8 = 512$.

Operation, 8)8192

8)1024

8) 128

Answer, 16

26. If 105 yards of broad cloth cost $735, how many dollars does 1 yard cost? Ans. $7.

DIVISION AND MULTIPLICATION.

1. A piece of cloth 13 yards in length was sold at auction for $130, what is the price of 5 yards? Ans. $50.

ILLUSTRATION.

By division we can ascertain the price of 1 yard, and by having the price of 1 yard, we can determine the price of 5 yards by multiplication.

Operation, 13)130(10 1 yard cost $10
13 5
0 $50 Ans.

Note to teachers.—Require the learner (as in the above example) to find the price of 1 yard by division, and by multiplication, to find the value of any given number of yards.

2. If 24 yards cost $96, what is the price of 9 yards? Ans. $36.
3. If 9 yards cost $36, what will 24 yards come to? Ans. $96.
4. If 8 yds. cost $32, what will 5 yds. cost? " 20.
5. If 5 yds. cost $20, what will 8 yds. cost? " 32.

MENTAL OPERATIONS.

1. If five pounds of coffee cost 75 cents, what is the cost of 2 pounds? Ans. 30c.

2. If 8 pounds of sugar cost 72 cents, how much did 3 pounds cost? Ans. 27c.

3. If 8 yards domestic muslin cost 96 cents, how much did 7 yards cost? Ans. 84c.

4. If 7 yards cost 84 cents, how much did 5 yards cost? Ans. 60c.

5. If 5 yards of broad cloth cost $15, how much will 9 yards cost at the same rate? Ans. $27.

6. If 9 yards cost $27, how much did 5 yards cost? Ans. $15.

7. If 5 pounds of butter cost $1, how much did 3 lbs. cost? Ans. 60c.

8. If 3 bushels of corn cost $1.50, how much did 2 bushels cost at the same rate? Ans. $1.

EXERCISE QUESTIONS.

1. A merchant paid $11.25 for a quantity of lemons, at 5 cts. a piece, how many lemons did he buy? Ans. 225

2. A farmer sold a quantity of pork for $30, at 8 cents per lb., how many pounds did he sell? Ans. 375.

3. A farmer sold a quantity of cheese for $29.82, at 7 cts. per lb., how many lbs. did he sell? Ans. 426.

4. How many oranges can you buy for $44.48, if you pay 8 cents a piece? Ans. 556.

5. A drover bought 12 oxen for $636, how many dollars did he pay for each? Ans. 53.

6. A gentlemen had 65 men in his employment to whom he paid for wages due $7,735, how many dollars had each man for his portion?

Operation, 65)7735(Here we find that the figure in the units place, in the divisor and dividend is 5, which is a *token*, for a common measure; hence, 13 × 5 = 1547 × 5, cancel the 5 from both sides, and it is reduced to 13)1547 = 119 quotient.

LONG DIVISION.

Division is a compendious method of subtraction, it teaches how often one number is contained in another. The number to be divided is called the *dividend*, the number to divide by is called the *divisor*, and the number of times the dividend contains the divisor, the *quotient*.—Sometimes there is a *remainder* left after the division is finished.

Q. How are the terms placed?

A. The dividend is the middle term. The divisor is placed on the left hand side, and the quotient on the right.

Q. In what manner?

A. By curved lines each side of the middle term or dividend, as to divide 12 by 4, the quotient is 3.

Dollars and cents may justly be considered as whole numbers and decimals, and are written as such.

EXAMPLE 1.

Divisor. Dividend. Quotient.

Divide ☞ 37(\$9869.75(\$266.75

74

246
222

249
222

277
259

185
185

To illustrate the above example ascertain by trying how often the left hand figure ☞ (3) of the divisor is contained in the left hand figure ☞ (9) of the dividend; we say 3 into 9 (3 times, we then multiply 37 by 3,

which produce 111, and find 3 times too much; as, 111, is more than 98. Again, we try 2 (twice) and find $37 \times 2 = 74$, we then set down 74 under 98, and take the difference, this difference or remainder is placed underneath the line drawn between it and the *subtrahend* 74. This being done, we next bring down the figure 6, which is placed to the right of the remainder, and find the number 246 represented as a *new* dividend; as before, we try how often the *left* hand figure 3 of the divisor is contained in 24, the 2 left hand figures of the new dividend, and find by trying that 6 times will answer, we then place the figure 6, after the figure 2 in the quotient, and multiply 37 by 6, which produce 222, and place it under 246, draw a line under, and take their difference. We again bring down the next figure 9 which is placed to the right of the remainder 24, and the number 249 is represented as another new dividend. Again, we try how often the left hand figure ☞3 is contained in 24, the two left-hand figures of the new dividend, and find by trying as usual, that 6 times will answer—6 times 37 are 222, which subtracted from 249 leaves a remainder of 27, the next figure 7 is brought down and placed to the right of the remainder 27, and the number 277 is represented. We proceed in like manner until the operation is performed, and find the quotient to be 266.75, we then commence at the right hand or units place, and point out two figures, 75 for cents, because there are cents in the dividend, consequently, two figures must be pointed.

1. A gentleman has a yearly income of $75,920, how many dollars is that a day, there being 365 days in a year? Ans. $208.

Operation,

Divisor.	Dividend.	Quotient.
365)	75920(	208
	730	
	2920	
	2920	

Or thus: the units place in the divisor is supplied by the figure 5, and the units place in the dividend, by a cipher (0;) consequently, 5 will be a '*token*' for a divisor or common measure. Here $73 \times 5 = 365$, which are factors.

Now, 5)75920

73)15184(208 Quotient by short division as before.
146

584
584

Seventy-three, the divisor in the last operation is a *prime* number, and cannot be decomposed into factors.

Q. What is a prime number?

A. A prime number is that which can be measured only by itself, or a unit, as 7, 11, 13, 19, 5, 23, 31, 37, &c.

Q. What is a composite number.

A. A composite number is equal to the product of its factors or component parts as $28 = 7 \times 4$, $24 = 8 \times 3$.

PROPERTIES OF NUMBERS.

It is evident from analysis, that much time and labor can be saved, in the operation of numbers, and, that questions which admit of many figures, can be readily answered *orally*, with precision and accuracy, agreeably to the following general rules, which are certainly of great importance, in elucidating the principles of the science.

RULE.

1. Figure 2 will divide all even figures without a remainder.

2. Figure 3 will divide any sum, when the addition of the numbers can be divided by 3.

3. Figure 4 will divide any number of figures when the two last figures on the right hand, or the two last on the left can be divided by it without a remainder.

4. Figure 5 will divide any number of figures, if the last figure on the right hand be a 5 or a *nought* or cipher, (as 0.)

5. Figure 6 is a factor in all *even* numbers, which have the token of 3, or in other words, in any even number of figures, whose sum can be divided by 6, as 24864, (their sum make 24,) 6, will be a token.

6. Figure 7 has a variety of tokens.

Case 1.—For two and three figures in a number, as 21, 42, 63, 84, 105, 126, 147, 168, 189. When the left hand figure or figures, are double those on the right as above, 7 will divide without a remainder.

Case 2.—By adding or subtracting 7, 14, &c. to or from the left hand figures, the token would be obtained, as 168 + 7 = 175 or 189—7 = 182; in either case, 7 will invariably be a divisor.

Case 3.—When the left hand figure divided into the two right hand figures, produce 5 in the quotient, as 315, 525, 945, (as 3 into 15) 5 times, 5)25(5 times, 9)45(5 times, figure 7 will be a general divisor or token; or, if 14, 21, &c. be subtracted from any number, 7 will be a token, as 434, minus 14, leaves 420, which is divisible by 7.

Case 4.—For four figures in a number. When a division of the two left hand figures into the two right hand figures give a quotient of 5, as 1155, 7 will be a token; or, if the division of the right hand figures into the two left hand figures, give a quotient of 3, as 6622. Again, if two digits inclose two ciphers, as 1001 to 9009, 7 will be a token.

Case 5.—For five figures in a number. When one cipher is inclosed by two equal numbers, as 27027, 7 will be a token.

☞ It is really a fact, that 7, 11, and 13, are factors in those numbers, which have the token of one or two ciphers inclosed by two equal numbers or digits as 1001.

ILLUSTRATION.

Suppose it were required to multiply 485 by 11, 7, and 13, to a continual product, the answer can be obtained correctly without any calculation, by placing the same number on the side of it, thus: 485485, the product as required.

Case 6.—In any given number of figures the token of 7 may be easily discovered by a division of the figures. Suppose 12684, 31563, 42525, 331121, 6301155, by inspection, you can instantly discover the token in the three hand, and two right hand figures of the first. In the second number you see the token in the three left hand figures, also in the two right hand figures. In the third number you can readily discover the token, and in the last number it can be seen in the three left hand, and four right hand figures.

Rule 7.—Figure 8 will divide any number without a remainder, when the three right hand figures are divisible by it, as 1267848, &c.

Rule 8.—When any given number of figures added together, as 121032 or 102105, make 9, 9 will be a token; thus, in the first number the aggregate is 9, which can be divided by 9, and the second number in like manner. It must be observed, that every number divisible by 9, is also divisible by 3, but every number divisible by 3, is not divisible by 9.

Rule 9.—Figure 10 is a factor in all numbers, of which the last figure is a cipher.

Rule 10.—By the figure 7, we find in what numbers 11 is a factor.

Q. Has it a general token?

A. It has in all numbers, of which the figures subtracted from the right or left hand, will leave no remainder, as 3861.

ILLUSTRATION.—8 minus 3 leaves 5, and 6 minus 1 = 5, hence, 5 — 5 = 0 or 8 — 3 = 5, and 6 — 5 = 1, and 1 — 1 = 0.

Note.—If a figure cannot be subtracted from the next, add 11 to it, and proceed as before. Thus, as in the number 3267, thus: 2 + 11 = (13 — 3) = 10, in like manner with the right hand figures, 6 + 11 = (17 — 7) = 10, and 10 — 10 = 0. Q. E. D.

Rule 11.—Figure 12 is a factor in all numbers which have the tokens of 3 and 4, as 3 × 4 = 12. Figure 13 is a factor when the left hand figure or figures divided into the right hand figures, will give a quotient of 4.

Rule 12.—Figure 14 is a factor in all numbers which have the token of 7. Figure 15 is a factor in all numbers which have the tokens of 3 and 5.

Rule 13 —Figure 25 is a factor in all numbers of which the two right hand figures divide without a remainder, as 1625, 3450, 7875, or a number which has two ciphers received on the end by a multiplication of 4.

Rule 14.—Figure 125 is a factor when the three right figures can be divided by it, as 34125, 7250, 89375, or when three ciphers are received on the end by a multiplication of 8. Multiply 4812 by 25, in the multiplicand we discover the token of 25 × 4 = 100, or thus:

Dividing 4)4812

1203 × 100 = 120300.

The product of any number to be multiplied by 25 may be obtained thus: place two ciphers at the end of the given number as 481200, which being divided by 4 will give 120300 the answer. Again, divide 789467125 by 125, agreeably to Rule 14, 125 is a token. Here 125 × 8 = 1000, hence, multiply the given number by 8, rejecting three places, and we have the quotient thus:

789467125

Multiply 8

6315737(000

From the following easy multipliers and divisors, the labor of *one hour can be done in a minute*, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 31, 41, 51, 61, 71, 81, 91, 100, 101, 901, &c., 10000, 100001, 900001, &c.

PRELIMINARY REMARKS.

If we multiply any number by 10, 100, 1000, &c., we place the number of ciphers to the product. If we multiply by 11, we place the figures of the multiplicand one place to the right or left hand under it, and add for the product. Thus:

Multiply 7683 by 11

7683

84513 by placing to the right,

or 7683
11

7683
7683

84513 common method by placing to the left.

If we multiply by 15, 14, &c., we multiply the multiplicand by the 5 or 4, and place the product to the right, as the 5 and 4 stand on the right hand of the unit, and add for the product.

SECOND METHOD.

as, 7365 by 14
29460

103110

or, 7365
14 common way.

29460
7365

103110

Thus you see, that the first method is the shortest.

If we multiply by 101, 102, 901, &c., we place the product two places to the right or left hand, as the figure of the multiplier stands, and add for the product.

Thus: 3465 by 106
20790

367290 The multiplier is to the right of the unit.

Again: 3465 by 701 multiplier to the left of the unit.
24255

2428965 product.

ANALYSIS OF NUMBERS.

Suppose it were required to analyse the number 3168, to its prime factors by inspection, agreeably to Rule 8, 9 is a token or factor, from whence we obtain $3168 = 9 \times 352$, these factors again analysed, we get $3 \times 3 \times 4 \times 88$, because, in 352 we discover the token of 4, agreeably to Rule 3, $3 \times 3 \times 4 \times 88$; 3×3 are prime numbers and cannot be analysed. We shall analyse 4×88, 4 is a token, $1 \times 4 = 4)4 \times 22$, expunge 4 from both sides and what remains but 22; in this we find that 2 is a token, and we have 2×11, the prime factors are 3, 3, 2, 2, 2, 2, 2, 11.

I have heretofore remarked, that agreeably to this plan, a person could in *one minute* perform a calculation which would require *one hour's* work in the ordinary manner to do it correctly. The following multiplication of factors will test the fact:

EXERCISE FOR THE SLATE.

Multiply 73678964 by 1429, 77, 28, 26, 25 and 5, continually, and tell the product.

Ans. 737747687568892000.

ILLUSTRATION BY ANALYSIS.

Multipy 73678964 by 1429, 77, 28, 26, 25, 5.

Analysing 77 to 7×11, 28 to 7×4, 26 to 13×2. Multiplying 25 by 4, we have 2 ciphers, and 5 by 2, we get 1 cipher more. The operation stands thus:

73678964 by 1429, 7×11, 4×7, 2×11, 100, 10.

By multiplying $1429 \times 7 = 10003$, in this number we find the tokens of 7, 11, and 13, and the work thus multiplied, we find the factors 7, 11, 13, remaining, which are tokens of 1001 product.

73678964 by 10003
221036892

737010676892 × 1001
737010676892

737747687568892000 Answer.

New factors 10003
Multiply 1001

10013063 Product.

Here by analysis the whole multiplication may be performed by a single multiplication of 3, almost in an instant. Multiply 931 by 325, 44 and 7 continually, and find the new factors and product. Ans. The factors are 13, 7, 11, 10, product, 93193100. The above continual multiplication may be performed by inspection, and the calculation verified for its certainty, in an instant.

ANALYSIS OF MULTIPLICATION AND DIVISION.

EXAMPLE.

Calculate the amount of 29 yards of muslin at 22 cts. per yard. Ans. $6.38.

ILLUSTRATION.

From the multiplication table 29 may be analysed thus: $2 \times 10 = 20 + 9$, that is, 2 tens and 9 over, and in like manner $22 = 2 \times 10 = 20 + 2 = 2$ tens and 2 over. Hence, from the properties of Euclid. 4, Lib. 2. "If a straight line be divided into any two parts the square of the whole line is equal to the squares of the two parts, together with twice the rectangle contained by the parts." Thus, let A $\overset{6}{\text{———}}\overset{4}{\text{———}}$ B be divided into two parts, 6 and 4, the square of the whole line 10, which is

equal to 6 squared plus 4 squared, together with twice the rectangle or product of $6 \times 4 = 24 \times 2 = 48$; hence, $10^2 = 6^2 + 4^2 + 2)6 \times 4 = 100 = (\overline{36 + 16 + 48}) = 100$.

dolls. dimes. cents.
First, 2 + 9 + 9 Multiply 2 by 2, and call the
2 + 2 + 2 product dollars, viz: $4.00
Add 9 and 2 the sum is 11, which multiply by
2, the product is 22 dimes, or - - 2.20
Again, 9 cents multiplied by 2, the product is 18

Proof by the common method $29 \times 22 =$ $6.38

Calculate the amount of 87 yds. at 83 cts.

"	"	73	"	85	"
"	"	35	"	38	"
"	"	95	"	97	"

— $239.71

CASE II.—*When the figures under the head of dollars are different.*

Example 1. Find the amount of 35 yards at 87 cents, - - - - - $24.00

dolls. dimes.
$3 \times 7 = 21$ $3 + 5 + 5$
$8 \times 5 = 40$ $8 + 7 + 7$

Sum, 61 dimes. 61 dimes is 6.10
Then 7 cents × by 5 cents, is - - .35

$30.45

2. Calculate the amount of 83 yds. at 63 cts.

"	"	54	"	35	"
"	"	27	"	49	"
"	"	99	"	39	"
"	"	56	"	78	"

— $166.71

3. If 24 yards of cloth cost $43,92, the price of 1 yard is required? Ans. $1.83.

Q. If there be cents or decimals in the dividend, and none in the divisor, how do you proceed in pointing?

A. Point out as many figures in the quotient as there are cents or decimals in the dividend.

Q. If there be as many cents or decimals in the divisor, as there are in the dividend, how do you proceed?

A. The quotient will be expressed in whole numbers in that case, there being no difference between the decimals in the divisor and dividend.

Q. If there should be more decimal places in the divisor than in the dividend. For instance, three decimal places in the divisor, and two places in the dividend?

A. One or more ciphers may be added to those in the dividend. Then, the difference between the decimal places in the divisor and dividend will show the number of decimal places in the quotient when pointed.

DIVISION OF FEDERAL MONEY.

Divide	140 dollars 85 cts.	by	15	quotient	$9.39
"	485 "	"	5	"	97.00
"	694.42*c.*2*m.*	"	1238	"	56.90
"	156 "	"	4	"	39.00
"	10368 "	"	36	"	288.00
"	2688 "	"	112	"	24.00
"	101442075 "	"	4025	"	25203.00
"	403524396 "	"	44	"	83372.00

QUESTIONS FOR THE SLATE.

Divide	$1984911.18¾	by	$44.25	Ans.	$44856.75
"	4640.18¾	"	15.00	"	309.34½
"	7550.00	"	125.00	"	60.40
"	58.14	"	38.00	"	1.53
"	1947.22	"	44.25*c.*5*m.*	"	44.50
"	1562.67	"	64.44	"	24.25
"	120.99	"	22.20	"	5.45
"	49561776.00	"	5137.00	"	9648.00
"	2748900.00	"	350.00	"	7854.00
"	3250000.00	"	520.00	"	62500.00

Q. When there are ciphers in the dividend and divisor, how is the operation abbreviated?

A. By cutting off, or expunging from the end of each as many ciphers, as are contained in the one that has the least number.

Q. Divide 2150596750 by 125? Ans. 17204774.

In this case, 5 is a token for a divisor, agreeably to question 11; consequently, $125 = 25 \times 5$, but 25 is a composite number, then, 5×5 are its factors, and we get $5 \times 5 \times 5 = 125$.

5)2150595750 Having only 30 figures in the operation, exclusive of the answer.

5)430119350

5)86023870

17204774 Answer.

By the common method of division it will be found that there are 52 figures in the operation, exhibiting a difference of 22 figures!! So much for *lengthy, protracted, old-fashioned methods.*

APPLICATION.

1. Bought 64 yards of cloth for $496, what was it per yard? Ans. $7.75.

2. Sold 27 bushels of wheat for $33.75, how much was it per bushel? Ans. $1.25.

3. If 13 pounds of butter cost $3.51, what was it per pound? Ans. 27c.

4. If 19 barrels of cider cost $42.75, what was it per barrel? Ans. $2.25.

5. If 35 hogsheads of molasses cost $551.25, what was it pr. gallon? (allowing 63 gals. in a hhd.) Ans. 25c.

6. Bought 3 pieces of domestic muslin, each piece containing 29 yards for $13.05, what was it per yard? Ans. 15c.

7. Bought 12 bushels of apples for $7.80, how much did 1 bushel cost? Ans. 65c.

ADDITION AND SUBTRACTION,

1. If I add 60, 90, and 560, and subtract from their amount 30, 40, and 90, what number will remain?

Ans. 550.

2. If in 20 chests, be 20 drawers, in every drawer be 20 purses, in every purse $20, how many purses and dollars do the 20 chests contain? Ans. 400 purses, and $8000.

ADDITION AND MULTIPLICATION.

1. There are 10 bags of coffee, weighing each 160 lbs., and 12 bags weighing each 135 lbs., what is the weight of the whole? Ans. 3220 lbs.

SUBTRACTION AND MULTIPLICATION.

1. There are 15 bags of coffee, each weighing 112 lbs., the bags which contain the coffee weighing 30 lbs., how much would the coffee weigh without the bags?

Ans. 1650 lbs.

LECTURE VI.

ON DECIMALS.

Whoever may have been the inventor of *Decimals*, Moses, the Lawgiver of the Jews, the Cotemporary of Cadmus, appears to have been among the number of those, who first employed them to any considerable extent. Independent of any extraordinary divine influence upon the mind of this individual, he was as a man, one of the first, during the times in which he lived. He was educated at the Court of Pharoah, "in all the learning of the Egyptians," which was, of course, in all the sciences then known. This learning was necessarily communicated in connexion with the Egyptian system of hieroglyphics; therefore, he was aware, of the difficulties his countrymen had to encounter, in relation to their land marks being defaced by the ebbing and flowing of the river Nile, and in consequence thereof, decreed, that each individual should provide for his own subsistence, without seizing what was in the possession of another; hence, *necessity and self-preservation* gave rise to the useful arts. It was then, the science of GEOMETRY became known. It was then, that man beheld with new eyes the magnificent spectacle, which nature exhibited to his senses and imagination. It was then, he learned to compare and examine; and it was then, his ideas were transported, as it were, into an *intellectual* world. He studied the phenomena of nature with discriminating attention, till his mind was impressed with a desire to know the *causes* by which they were produced. From the history of Jose-

phus, we have been informed that the Patriarch Abraham, retired from CHALDEA into Egypt, during the time of a famine, and taught the inhabitants of certain portions of that country a knowledge of arithmetic. Also, that his mode of instruction was based upon the method of Notation, substituting Hebrew letters for modern numerals, as *Aleph* 1, *Beth* 2, &c. to *Yod* 10, *Caph* 20, to *Keph*, 100, &c., reckoning singly from 1 to 10. So that it is evident, he understood the *decimal scale* or decuple division of numbers: these facts and their resulting influence are traced as far as human knowledge extends; in short, they are from the very time of the invention of letters. Hence, it is natural to suppose, that some knowledge of numbers more or less perfect, must have been coeval with human society, from the bare consideration of our natural wants and early impressions.

THE DENARY OR DECIMAL SCALE OF TENTHS.

Q. What is a decimal fraction?

A. It is a fraction having always some power of 10, for its denominator.

Q. When an integer or whole is supposed to be divided into any given number, as $\frac{2}{10}$, $\frac{3}{100}$, $\frac{5}{10000}$, &c., how is it expressed or written?

A. As .2 .02 .002, &c. with a point (.) to the left.

Q. What is the general rule in this case?

A. To point off as many figures as there are ciphers in the denominator; the places between the significant figure and the point being supplied with ciphers if necessary, as above. Consequently, the same number of figures on the right of the decimal point, has always the same denominator.

Q. How is the value of digits ascertained agreeably to the *law of increase* from the Denary Scale.

A. From Notation we know the value of figures *increases* in a ten-fold proportion from the right to the left.

Thus: thousands. hundreds. tens. units.

$$1111 = 1000 + 100 + 10 + 1 = 10^3 + 10^2 + 10 + 1;$$

hence, it is evident, that the distance of any figure from the right indicates the power of 10.

Q. How is the value of decimal figures ascertained agreeably to the *law of decrease?*

A. In mixed numbers the decimals are separated from the integers by a point; thus: $25\frac{2}{100}$ is written 25.02, agreeably to question 4. "We point out as many decimal places as there are ciphers in the denominator." Consequently, it can be easily perceived that the value of decimals decreases in the same ten-fold proportion from the point (.) towards the right hand, as that of *integers*, increases from the point towards the left.

In GEOMETRY we find that lines emanate or flow from a point (.); so in ARITHMETIC, the power of numbers is derived from a unit, and although we can by the preceding rules apply to fractions in all cases. The four fundamental operations of arithmetic, yet it must have been long since perceived, that if the different sub-divisions of a unit employed for measuring quantities smaller than the unit, had been subjected to a common law of decrease the *calculus* of fractions would have been much more convenient on account of the facility with which we might convert one into another. By making this law of decrease conform to the basis of our system of numeration, we have given to the *calculus* the greatest degree of simplicity of which it is capable.

ILLUSTRATION.

Now as a unit is conceived to be divided into 10 equal parts, each part would be a *tenth*, and if each of these

parts or *tenths* were again divided into 10 parts, each would be one-hundreth part of a unit. Again, if these were again divided, in like manner, each part would be one-thousanth part of a unit and so on.

Example, 1 cent or $\frac{1}{100}$ part of a dollar is expressed thus: .01, agreeably to question 4; because, we must point out two places for cents invariably.

NUMERATION OF DECIMALS.

EXAMPLES.

Write down a unit?			$1.00	Unit.
Write down	$\frac{1}{10}$	decimally?	0.10	Hundreths.
" "	$\frac{1}{100}$	"	0.01	Hundreth.
" "	$\frac{1}{1000}$	"	0.001	Thousanth.
" "	$\frac{1}{10000}$	"	0.0001	Tenths of thousands.
" "	$\frac{1}{100000}$	"	0.00001	Hundreths of thousands.
" "	$\frac{1}{1000000}$	"	0.000001	Millionth of a unit or dollar.
" "	$\frac{1}{10000000}$	"	0.0000001	10 Millionth part of a dollar.
" "	$\frac{1}{100000000}$	"	0.00000001	100 Millionth part of a unit or dollar.

ADDITION OF DECIMALS.

(Sign +)

RULE.—Add as in whole numbers.

Add	65.43.9	Add	400.03.9	Add	4.00.30
"	84.73.6	"	640.72.6	"	54.97.40
		"	640.65.4	"	3.21.00
	150.17.5	"	947.67.6	"	6.72.03
					68.90.73

Add	647.83.9	Add	43.00.7	Add	427.00.0
"	494.74.7	"	64.73 8	"	603.04.0
"	648.93.5	"	79.89.4	"	210.15·0
"	943.49.7	"	38.49.6	"	3.36.
"	846.79.9	"	44.48.4	"	0.02.
"	469.94.6				
					1243.57.0

MULTIPLICATION OF DECIMALS.

(Sign ×)

RULE.—Point off as many decimals in the product as are in both Factors.

1.	Multiply	231.415	by	8	Ans.	1851.320
2.	"	32.1509	"	15	"	482.2635
3.	"	.840	"	840	"	705.600
4.	"	1.236	"	13	"	16.068
5.	"	223.86	"	2.500	"	559.6500
6.	"	35.640	"	26.18	"	993.05520
7.	"	8.4960	"	2.618	"	22.2425280
8.	"	0.5236	"	0.2808	"	0.14702688
9.	"	0.11785	"	.27	"	0.03181950

CALCULATION OF BILLS BY DECIMALS.

J. Thompson Laws, Bought of John Jones,

17 yds. of flannel	at	47 cts.	
19 " shalloon	"	37 "	
16 " blue camlet	"	46 "	
13 patterns silk vestings	"	3.78 "	
9 yds. cambric muslin	"	63 "	
25 " bombazine	"	56 "	
17 " ticking	"	31 "	
19 " striped jean	"	16 "	
			$99.50

EXERCISES FOR THE SLATE.

1. What will 126 lbs. butter cost at 12c. pr. lb? $15.12
2. What will 63 lbs. tea cost at 65c. per lb? 40.95
3. What will 13 tons of hay cost at $15.75 pr. ton? 204.75
4. What will 46 lbs. pork cost at 7c. 5m. pr. lb? 3.45
5. What will 76 cwt. beef cost at $5.28 pr. cwt? 401.28
6. What will 19 cords of wood cost at $3.75 pr. c? 71.25
7. What will 769 lbs. of leather " .195 pr. lb? 149.95½
8. What will 65 gals. of wine " $1.75 pr. gal? 113.75
9. What will 796 lbs. of cocoa " $6.71 pr. lb? 5341.16
10. What will 19 bbls. of cider " $1.37½ pr. bbl. 26.12½
11. What will 49 green hides " $1.95 95.55
12. What will 15* gross bottles of castor oil cost at .375 per bottle? - - - 810.00
13. What will all the above items amount to? $7273.34

CASE I.

RULE 1.—If the number of figures in the product is not so great as the number of decimals in the multiplier and multiplicand, a sufficient number of ciphers must be placed to the left of the product, to make the figures in the product equal to the number of decimals in both factors, and the decimal point must then be placed to the left of the ciphers, as in example 4th.

EXAMPLE 1.

1. Multiply - - - .18 by 07
   .07

   Agreeably to Case 1, Rule 1, .0126
2. Multiply .008 by .08.
3. Multiply .0008 by .008.
4. Multiply $1.05*c*.3*m*. by one dollar and 2 cents.
5. Multiply .50 cents by .50 cents. Ans. 25c.
6. Multiply .10 " by .10 " " 1c.

* 144 or 12 dozen = gross.

7. Multiply .60 cents by .60 cents. Ans. 36c.
8. " .25 " by .25 " " 6¼c.
9. " .75 " by .75 " " 56¼c.
10. " .40 " by .40 " " 16c.
11. " 1 cent by 1 cent. " .0001c.
12. " 5 cents by 5 cts. A. ¼ of a ct. or .0025
13. " .20 " by 20 " Ans. 4c.
14. " .30 " by .30 " Ans. 9c.
15. " .50 " by .75 " Ans. 375 or 37½c.
16. " .345 " by 6.25 cts. Ans. 2.15625
17. " 5 mills by 5 mills. " 0.000025

QUESTIONS FOR THE SLATE.

Multiply	.18	by	.24	Ans.	4.32
"	36.1	by	2.5	"	90.25
"	54.2	by	38.63	Ans.	2093.746
"	4560.	by	.372	"	1696.320
"	.28043	by	.0005	Ans.	.000140215
"	.00071	by	.121	"	.00008591
"	4.001	by	.004	"	.016004

SUBTRACTION OF DECIMALS.

(Sign —)

The rules prescribed for the subtraction of whole numbers apply also to decimals.

From	4562	From	86.746	From	978.4464
Take	3160	Take	73 638	Take	435.4969
	1402				

From	9648.6978	From	986974.668875
Take	4697.6986	Take	745649.954689

1. From $100, take 1 mill? Ans. $99.99*c*.9*m*.
2. From $400, take 1 cent? Ans. $399.99
3. From $750, take $5.05.5? " $744.94.5
4. From $110, take $4.44.4? " $105.55.6

5. From $50, take five dollars, five cents and five mills?

6. From $80, take $1, one cent and 9 mills?

7. From $90, take $1, nine cents and 3 mills?

8. From seventy-five dollars, seven cents and 7 mills, take nine dollars, nine cents, and nine mills?

DIVISION OF DECIMALS.

(Sign ÷)

Division of Decimals is performed the same as in whole numbers, only observing, that the number of decimals in the quotient must be equal to the excess or difference between the number of decimals of the dividend, and those of the divisor. When the divisor contains more decimals than the dividend, ciphers must be added or affixed to the right hand of the latter, to make the number equal to, or exceed, that of the divisor.

Example 1.—Divide 14.625 by 3.25.

Divisor. Dividend. Quotient.

3.25)14.625(4.5
1300

1625
1625 Answer.

In this example there are 2 decimals in the divisor, and 3 in the dividend, their difference 3 — 2 = 1; hence, only 1 decimal is to be pointed off in the quotient.

Example 2.—Divide 3.1 by 0.0062.

Q. Which is the divisor?

A. The number to the right of the word '*by*' when set down, as, 0.0062.

Q. Which is the dividend?

A. The number to the left of the word '*by*,' as 3.1.

Q. How is the operation performed?

A. The same as in division of whole numbers.

Q. When there are more decimals in the divisor than in the dividend, how do you proceed?

A. By prefixing as many or more ciphers (as the case may be) to the right of the dividend as there are decimals in the divisor.

Divisor. Dividend. Quotient.

Illustration, .0062)3.100000(500.00

3.10

0000 Ans.

From this operation, the learner can see there are 6 places of decimals in the dividend, and 4 in the divisor, and by taking 4 from 6, the remainder is 2, which shows the number of decimal figures to be pointed off in the quotient.

CASE I.—*When there are as many decimals in the divisor as in the dividend.*

Q. How is the quotient expressed?

A. In whole numbers.

Give an example.

Divide 9.6 by .06.

Here, by prefixing a cipher to 9.6 it becomes 9.60, and then, has 2 decimals, the same as in the divisor 06)9.60(160, consequently, 160, is a whole number.

1.	Divide 17.256	by	1.16	Ans. 14.8750
2.	" 148.630	by	4.21.	" 35.304 +
3.	" 2142.	by	3 2	" 66.9375
4.	" 2.00385	by	931.	" 0.0021523
5.	" 64.395	by	40.5	" 1.59
6.	" 5.8674	by	127.	" 0·0462
7.	" 2033.100	by	0.324	" 6275.—
8.	" 1383.2	by	60.8	" 22.75

APPLICATION OF DIVISION AND MULTIPLICATION.

1. Divide $1 by five cents?
2. Divide $5 by 5 mills?
3. Divide $44.44 by 11 mills?
4. Divide $44.50 by 5 mills?

5. If 12 yards of cloth cost $4.50, how much did 1 yard cost? Ans. .37.5 or 37½c.

6. If 4 yards cost $1.25, how much did 3 yards cost? Ans. .9375 or 93¾c.

7. If 5 yards cost .62.5, how much did 8 yards cost? Ans. $1

8. If 7 yards cost .87.5, how much did 2 yards cost? Ans. .25

9. If 3 yards cost .37.5, how much did 7 yards cost? Ans. .875

10. If 7 yards cost .4375, how much did 16 yards cost? Ans. $1

11. If 16 yards cost $1, how much did 3 yards cost? Ans. .1875

12. If 5 yards cost .3125, how much did 8 yards cost? Ans. 50c.

13. If 3 yards cost .1875, how much did 9 yards cost? Ans. .5625

14. If 8 yards cost 50c. how much did 12 yards cost? Ans. 75c.

REDUCTION OF DECIMALS.

From what has been said, the following questions and answers are sufficiently evident, in relation to reduction of decimals.

CASE I.

Q. How is a number of a lower denomination changed to the decimal of a higher?

A. We first suppose it to be changed into a fraction, having 10, or some multiple of 10 for its denominator.

Q. How do you proceed next?

A. We add ciphers to the numerator, and divide it by so many as make one of this higher denomination, and the quotient is the required decimal.

Q. Can the decimal thus obtained be again converted into a decimal?

A. Yes, agreeably to the *decimal scale* it has 10, or a multiple of 10 for its denominator, and by division it can be reduced to a still higher name.

CASE II.—*To reduce a decimal of a higher name to a lower.*

Q. How is this kind of reduction performed?

A. We multiply the decimals by so many as make one of a lower denomination, and point off the decimals as already observed in multiplication. The figures which remain on the left of the period, when the proper number is separated for decimals will constitute, the whole number of this denomination.

Q. Can the decimal part be still reduced?

A. Yes, if there be lower denominations, multiply it by the number which is equal to one of the next denomination, and proceed as before.

AVOIRDUPOIS WEIGHT.

CASE I.—*To bring a lower denomination to a higher.*

1. Reduce 8 cwt. to the decimal of a ton.

ILLUSTRATION.

The highest denomination mentioned is *a ton*, which is 20 cwt. then the fraction will be expressed thus:

$4\left|\frac{8}{20}\right.$ or $\frac{8}{20}$, agreeably to question 8, annex two ciphers to the numerator $\frac{800}{20} = .40$ Ans.

2. Bring 7 cwt. to the decimal of a ton? Ans. .35
3. Bring 5 cwt. to the decimal of a ton? " .25
4. Bring 14 lbs. to the decimal of a cwt? " .125
5. Bring 21 lbs. to the decimal of a cwt? Ans. .1875
6. Bring 3 quarters to the decimal of a cwt. " .75
7. Bring 2 qrs. 14 lbs. to the " of a cwt. " .625
8. Bring 5 ounces to the decimal of a lb. " .3125

CASE II.—*To reduce a decimal of a higher name to a lower.*

1. Bring .4 of a ton to its proper value?

20

8.0 Answer 8 cwt.

2. Reduce .35 of a ton to its proper value? Ans. 7 cwt.
3. Reduce .25 of a ton to its proper value? " 5 "
4. Reduce .125 of a cwt. to its proper value? " 14 lbs.
5. Reduce .1875 of a cwt. to its proper value? " 21 "
6. Reduce .75 of a cwt. to its proper value? " 3 qrs.
7. Reduce .625 of a cwt. to its proper value?

Ans. 2 qrs. 14 lbs.

8. Reduce .3125 of a lb. to its proper value? Ans. 5 oz.

APOTHECARIES WEIGHT.

CASE I.—*Bringing a lower number to the decimal of a higher.*

1. Bring 8 ounces to the decimal of a pound?

Example 4 | 8 = $\frac{8}{12}$ reduce $\frac{8}{12}$ to a decimal add two ciphers to the numerator $\frac{800}{12}$ = .66 +.

2. Reduce 2 scruples to the decimal of a pound?

Ans. .00794

3. Reduce 18 grains to the decimal of an ounce?

Ans. .0.0375

LAND MEASURE.

CASE I.

1. Reduce 20 perches to the decimal of an acre?

Ans. .125

2. Reduce 32 perches to the decimal of an acre?

Ans. .20

3. Reduce 16 perches to the decimal of an acre?

Ans. .0.10

4. Reduce 2 roods, 32 perches, to the decimal of an acre? Ans. .7

5. Reduce 3 roods, 20 perches to the decimal of an acre? Ans. .875

6. Reduce 3 roods, 35 perches to the decimal of an acre? Ans. .96875

CASE II.

1. Bring .125 decimal of an acre to its proper value? Ans. 20 perches.

2. Bring .20 of an acre to its proper value? Ans. 32p.

3. Bring .60 of an acre to its proper value? Ans. 2 roods, 16 perches.

4. Bring .96875 of an acre to its proper value? Ans. 3 roods, 35 perches.

5. Bring .45 of an acre to its proper value? Ans. 1 rood, 32 perches.

6. Bring .40 of an acre to its proper value? Ans. 1 rood, 24 perches.

CLOTH MEASURE.

CASE I.

1. Reduce 2 quarters and 2 nails to the decimal of a yard? Ans. .625

2. Reduce 3 quarters and 3 nails to the decimal of a yard? Ans. .9375

3. Reduce 1 quarter and 1 nail to the decimal of a yard? Ans. .3125

CASE II.

1. Bring .625 of a yard to its proper value? Ans. 2 grs. 2 nails.

2. Bring .9375 of a yard to its proper value? Ans. 3 qrs. 3 nails.

3. Bring .3125 of a yard to its proper value? Ans. 1 qr. 1 nail.

4. Bring .865 of a yard to its proper value? Ans. 3 qrs. 2 nails.

5. Bring .375 of a yard to its proper value? Ans. 1 qr. 2 nails.

APPLICATION.

1. What will 3.875 acres come to at $10 per acre?
Ans. $38.750
2. What will 15 125 acres cost at $30.75 per acre?
Ans. $465.093
3. What will 10.625 cwt. of iron cost at $5.25 pr. cwt?
Ans. $55.781
4. What will 10.1875 cwt. " cost at $5.35 per cwt?
Ans. $54.503
5. What will .125 cwt. " cost at $7.10 per cwt?
Ans. .887
6. What will .3125 lbs. of tea come to .87*c*.5*m*. per lb?
Ans. .273
7. What will .35 of a ton come to at $96.50 per ton?
Ans. 33.775
8. What will .4 of a ton cost at $95 per ton?
Ans. $38.00
9. What will .625 of a yard cost at $4.50 per yard?
Ans. $2.812
10. What will .875 of a yard cost at 37*c*. 5*m*. per yard?
Ans. .328
11. What will .8 of an English Ell cost at $3.25 pr. yd?
Ans. $3.25
12. What will .1875 of a yard cost at $5.00 per yard?
Ans. .937
13. What will .375 of a ton cost at $80 pr. ton? " $30
14. What will .859375 bushels come to at $1.3125 per bushel? Ans. $1.128
15. What will .875 lb. avoirdupois cost at 15c. per lb?
Ans. .131
16. What will 3.75 gal. cost at 33c. pr. gal.? " 1.237
17. What will .125 gals. .375 " " .047
18. What will 333.75 ft. of boards come to at 3c. pr. ft?
Ans. $10.012
19. What will $33\frac{1}{12}$ dozen buttons cost at 87*c*. 5*m*. per dozen? Ans. $29.385

LECTURE VII.

ON REDUCTION.

Reduction admits of MULTIPLICATION and DIVISION, and of course, the signs $\times$ and $\div$ are understood, when used in reference to these rules.

> REDUCTION shows, how we of names in use,
> May *high* to *low* and low to high reduce;
> So that the answer, which shall thence arise,
> The given sum in value equalize.

Reduction, in other words, is an application of Multiplication and Division: when higher quantities are reduced to lower, as years to seconds, Multiplication is used and the operation is called *Reduction descending.*

ILLUSTRATION.

Suppose we bring 1 year to months; we know agreeably to the *law of increase,* that the operation is performed by multiplication; hence, we multiply by 12, to bring years to months. On the contrary, division is used in reducing lower names to higher, as when we want to reduce (lbs.) pounds to hundreds, (or cwts.) we divide the pounds by 28, to bring them to quarters, the quarters by 4, to bring them to cwts. This reduction is called reduction ascending.

FEDERAL MONEY, OR UNITED STATES CURRENCY.

The denominations are—

10 mills (m)	make	1 cent, -	- c.
10 cents (cts.)	"	1 dime, -	- d.
10 dimes	"	1 dollar,	- $ or D.
10 dollars	"	1 Eagle, -	- E.

AVOIRDUPOIS WEIGHT.

Every thing of a coarse drossy nature is bought and sold by this weight.

FIRST FORM OF THE TABLE.

Multipliers.	20 hundred weight make	1 ton,	T.
	4 quarters or 112 lbs. make	1 hundred,	cwt.
	28 pounds make	1 qr. of a cent,	qr.
	16 ounces "	1 pound,	lb.
	16 drams "	1 ounce,	oz.

Q. How are these numbers used?

A. As multipliers, when a higher name is reduced to a lower.

SECOND FORM.

Divisors.	16 drams make	1 ounce,	oz.
	16 ounces "	1 pound,	lb.
	28 pounds "	1 qr. of a hun.	qr.
	4 quarters "	1 hundred,	cwt.
	20 hundred weight make	1 ton,	T.

Q. How are the numbers in the second form of the table of Avoirdupois weight used?

A. As divisors, when bringing a lower name to a higher.

CASE I.—*Bringing a higher name to a lower.*

Example 1. Bring 2 tons to drams? Ans. 1146880 drs.

Q. What are tons multiplied by?

A. By 20.

Q. Why?

A. Because 20 hundreds make a ton.

Q. What are cwts. multiplied by?

A. By 4.

Q. Why?

A. Because 4 quarters make 1 cwt. weight.

Q. What are quarters multiplied by?

A. By 28.

Q. Why?

A. Because 28 lbs. make a quarter.

Q. What are pounds (lbs.) multiplied by?

A. By 16.

Q. Why?

A. Because 16 ounces make a pound (lb.)

Q. What are ounces multiplied by?

A. By 16.

Q. Why?

A. Because 16 drams make an ounce, (oz.)

Q. What does the first form of the table refer to?

A. To reduction descending.

Q. Recite rule 1st?

A. Tons multiplied by 20 are cwts.

Cwts. " by 4 are quarters.

Quarters " by 28 are lbs.

Pounds " by 16 are ounces.

Ounces " by 16 are drams.

The learner is earnestly requested to recite the table of Avoirdupois weight, agreeably to Rules 1 and 2, as laid down in this work.

EXAMPLE.—Bring

tons.	cwt.	qrs.	lbs.	ozs.	drams.
1	17	3	17	14	13

to drams?

20

By Rule 1. 37 cwts.

4

151 quarters.

28

1215
303

4245 pounds.

16

25474
4246

67934

16

407607
67935

1086957 drams.

ILLUSTRATION.

First we multiply the tons by 20, and add '*in*' the 17 cwts. and we get 37 cwts. The cwts. we multiply by 4, and add in 3 quarters, which make 151 quarters; the quarters are multiplied by 28, and 17 lbs. are added, which make 4245 lbs. The pounds are multiplied by 16, and 14 ounces are added, which make 67934 ounces. Again, the ounces are multiplied by 16, and 13 drams are added; hence, we get 1086957 drams for the answer required.

APPLICATION.

1. Bring 13 tons to cwts? Ans. 260 cwts.
2. Bring 260 cwts. to quarters? Ans. 1040 qrs.
3. Bring 36 quarters to pounds? " 1008 lbs.
4. Bring 17 pounds to ounces? " 272 ozs.
5. Bring 20 ounces to drams? " 320 drs.
6. Reduce 3 quarters, 14 lbs. 11 ozs. 11 drs. to drams? Ans. 25275 drams.
7. Reduce 5 tons, 12 cwt. 2 qrs. to quarters? Ans. 450 qrs.
8. Reduce 2 qrs. 14 lbs. 10 ozs. to drs. " 18080 drs.
9. Reduce 12 cwt. 1 qr. 22 lbs. to lbs? " 1394 lbs.

Illustration.—To analyse 112, we get $100 + 12 = 112$.

Common method.

cwts.	qrs.	lbs.
12	1	22
4		
—		
49		
28		
—		
394		
100		
—		

1394 lbs. answer.

Oral Reduction of cwts. qrs. and lbs.

cwts.

$12 \times 100 = 1200$

$12 \times 12 = 144$

1 qr. and 22 lbs. $= 28 + 22 = 50$

Answer, 1394 lbs.

10. Bring 15 cwt. 3 qrs. 20 lbs. to lbs? Ans. 1784 lbs.
11. Bring 18 " 2 " 18 " to lbs? " 2090 "

12. Bring 38 cwt. 3 qrs. 20 lbs. to lbs? Ans. 4360 lbs.
13. Bring 56 " 2 " 14 " to lbs? " 6342 "

APOTHECARIES WEIGHT.

By this weight Apothecaries mix their medicines, they give only 12 ounces to a pound.

FIRST FORM OF THE TABLE.

The denominations are—

12 ounces	make	1 pound,	℔
8 drams	"	1 ounce,	℥
3 scruples	"	1 dram,	ʒ
20 grains (gr.)	make	1 scruple,	℈

Q. How are these numbers used?

A. As multipliers, when a higher name is reduced to a lower.

SECOND FORM.

20 grains	make	1 scruple,	℈
3 scruples	"	1 dram,	ʒ
8 drams	"	1 ounce,	℥
12 ounces	"	1 pound,	℔

Q. Recite Rule 1st?

A. Pounds multiplied by 12 are ounces.
Ounces " by 8 are drams.
Drams " by 3 are scruples.
Scruples " by 20 are grains.

CASE I.—*Bringing a higher name to a lower.*

Example. Rule 1st.—Bring 28 lbs. to ounces?

12

Answer, 386 ounces.

1. Bring 72 ounces to drams? Ans. 576 drs.
2. Bring 10 lbs. to grains? " 57600 grs.
3. Bring 8 lbs. to drams? " 768 drs.
4. Bring 28 drams to scruples? " 84 scruples.
5. Bring 72 scruples to grains? " 1440 grs.
6. Bring 15 lbs. 9 oz. 4 dr. 2 scr. to grs? " 91000 "

TROY WEIGHT.

First form of the Table.

By this weight, gold, jewels, silver and liquors are weighed.

12 ounces	make	1 lb.
20 pennyweights	make	1 oz.
24 grains	"	1 dwt. (pennyweight.)

Q. How are these numbers used?

A. As multipliers, when arranged agreeably to the first form of the table.

Second form of the Table.

24 grains	make (pennyweight)	1 dwt.
20 pennyweights	make	1 oz.
12 ounces	"	1 lb.

CASE I.—*Bringing a higher name to a lower.*

APPLICATION.

1. Bring 27 lbs. 10 ozs. 13 dwts. of gold, to grains? Ans. 160632
2. Bring 8 lbs. 0 oz. 7 dwts. 2 grs. to grains? " 46250
3. Bring 375 lbs. 10 ozs. 16 " " Ans. 2164816
4. Bring 29 ozs. 16 dwts. to dwts.? " 596
5. Bring 19 lbs. 11 ozs. 14 dwts. 21 grs. to grains? Ans. 115077
6. Bring 125 lbs. to grains? " 720000
7. Bring 59 lbs. 13 dwts. 5 grs. to grains? " 340157

LONG MEASURE.

This measure is used for measuring distances.

First form of the Table.

360 degrees is equal to the circumference of the earth.

69½	statute or geographic miles,	1 degree.
3	miles (league) - - - -	1 league.
8	furlongs (or 1760 yards,) - -	1 mile.
40	perches (or 220 yards,) - -	1 furlong.
5½	yards, - - - - -	1 perch.

3	feet,	1 yard.
12	inches,	1 foot.
3	barley corns,	1 inch.

Second form of the Table.

3	barley corns,	1 inch.
12	inches,	1 foot.
3	feet,	1 yard.
5½	yards,	1 perch.
40	poles or perches,	1 furlong.
8	furlongs,	1 mile.
3	miles,	1 league.
60	geographic or 69½ statue miles,	1 degree.

360 degrees the circumference of the earth.

Q. How do you bring miles to barley corns, agreeably to Rule 1?

A. Miles are multiplied by 8 to reduce them to furlongs,

Furlongs	"	by 40 are perches.
Perches	"	by 5½ are yards.
Yards	"	by 3 are feet.
Feet	"	by 12 are inches.
Inches	"	by 3 are barley corns.

CASE I.—*Bringing a higher name to a lower.*

APPLICATION.

1. Bring 273 miles to inches? Ans. 17297280 in.
2. Reduce 2 m. 1 ft. 8 p. 3 yds. 2 in. to inches? Ans. 136334 in.
4. Reduce 29 miles to inches? " 1847440 in.
4. Reduce 16 furlongs to poles? " 640
5. Reduce 70 miles, 7 fur. to furlongs? Ans. 567 ft.
6. Reduce 3 leagues to poles? " 2880 ps.
7. Reduce 2 yards, 3 inches to inches? " 75 in.
8. Reduce 17 yards, 1 foot to feet? " 52 ft.
9. Reduce 52 perches to yards? " 286 yds.

CLOTH MEASURE.

The denominations of Cloth Measure are English Ell, Flemish Ell, yard and nail.

First form of the Table.

6	quarters	make	1 Ell French.
5	"	"	1 Ell English.
4	"	"	1 yard.
3	"	"	1 Ell Flemish.
4	nails	"	1 quarter of a yard.

Q. How are the numbers used in the form of the above table?

A. As multipliers.

Q. When they are employed as multipliers, what is the reduction called?

A. Reduction descending.

Second form of the Table.

4	nails (na.)	make	1 quarter of a yard.
4	quarters	"	1 yard.
3	"	" (E. F.)	1 Ell Flemish.
5	"	" (E. E.)	1 Ell English.
6	"	" (E. Fr.)	1 Ell French.

CASE I.

APPLICATION.

1.	Bring 56 English Ells to quarters?	Ans. 280 qr.
2.	Bring 85 French Ells to quarters?	" 510 "
3.	Bring 91 Flemish Ells to quarters?	" 273 "
4.	Bring 25 yards to nails?	" 400 na.
5.	Bring 27 yards to quarters?	" 108 qr.
6.	Bring 35 quarters to nails?	" 140 na.

LAND OR SQUARE MEASURE.

The denominations of Land Measure, are acre, rood, square perch, and square yard.

First form of the Table.

4 roods	make	-	-	1	acre.
40 square perches	make		-	1	rood.
30¼ square yards	"	-	-	1	square foot.
9 square feet	"		-	1	" yard.
144 square inches	"	-	-	1	" foot.

Second form of the Table.

144 square inches	make	-	-	1	square foot.
9 " feet	"		-	1	" yard.
30¼ " yards	"	-	-	1	" perch.
40 " perches	"		-	1	rood.
4 roods		"	-	-	1 acre.

CASE I.—*Bringing a higher name to a lower.*

APPLICATION.

1.	Bring 35 acres to perches?	Ans.	5600p.
2.	Bring 30ac. 1rd. 10pr. to perches?	"	4850p.
3.	Bring 5ac. 3rds. 32pr. to perches?	"	952p.
4.	Bring 1 acre to square yards?	Ans.	4840 sq. yd.
5.	Bring 1 acre to square feet?	"	43560 sq. ft.
6.	Bring 15 roods to sq. perches?	"	600 perch.
7.	Bring 50 square yards to sq. ft?	"	450 sq. ft.
8.	Bring 5 square feet to sq. in.?	"	720
9.	Bring 16 sq. perches to sq. yds?	"	484
10.	Bring 3rds. 25pr. to sq. inches?	"	5684580 sq. in.

LIQUID MEASURE.

The denominations are gills, pints, quarts, gallons, hogsheads, and tuns.

4 gills	make	-	-	1 pint.
2 pints	"	-	-	1 quart.
4 quarts	"	-	-	1 gallon.

31½ gallons make - - - 1 barrel.
42 " " - - 1 tierce.
63 " " - - - 1 hogshead.
4 hogsheads make - - 1 tun.

Note.—1 pipe is equal to 2 hogsheads; therefore, 2 pipes is equal to 1 tun.

REDUCTION DESCENDING.

CASE I.

Tuns multiplied by 4 are hogsheads.
Hhds. " by 63 are gallons.
Gallons " by 4 are quarts.
Quarts " by 2 are pints.

1. Bring 17 quarts to pints? Ans. 34 pt.
2. Bring 25 gallons to quarts? " 100 qr.
3. Bring 5 hhds. to gallons? " 315 yds.
4. Bring 100 gallons to pints? " 800 pt.
5. In 6 tuns, how many pints? Ans. 12096 pt.
6. Bring 7 hhds. 41 gals. 2 qrts. to qrts? " 1930 qr.
7. Bring 47 gals. 2 qrts. to pints? " 380 pt.
8. Bring 4 hhds. 3 qrts. to pints? " 2022 "
9. Reduce 19 tuns, 27 gals. to quarts? " 19260 qts.
10. Bring 5 tuns, 1 hhd. 15 gals. to qts? " 10707 pts.

REDUCTION ASCENDING.

Gills divided by 4 are pints.
Pints " by 2 are quarts.
Quarts " by 4 are gallons.
Gallons " by 63 are hhds.
Hhds. " by 4 are tuns.

AVOIRDUPOIS WEIGHT.

CASE II.—*Bringing a lower name to a higher.*

Q. How is the operation performed?
A. By division.

Q. How are the divisors arranged?

A. Agreeably to the second form of the table of Avoirdupois weight.

APPLICATION.

1. Bring 1040 quarters to cwts? Ans. 260 cwt.
2. Bring 1008 pounds to quarters? " 36 qrs.
3. Bring 320 drams to ounces? " 20 ozs.
4. Bring 1130 ounces to a higher denomination?
Ans. 2 qrs. 14 lbs. 10 ozs.
5. Bring 540 cwts. to tons? Ans. 27 tons.
6. Bring 7000 lbs. to a higher denomination?
Ans. 62 cwt. 2 qrs.
7. Bring 180 quarters to tons? " 2 tons, 5 cwt.
8. Bring 640 drams to ounces? " 40 ozs.
9. Bring 1561 lbs. to cwts. qrs. and lbs?
Ans. 13 cwt. 3 qrs. 21 lbs.

APOTHECARIES WEIGHT.

CASE II.—*Bringing a lower name to a higher.*

Q. Recite the second form of the table.

A. Grains divided by 20 are scruples.
Scruples " by 3 are drams.
Drams " by 8 are ounces.
Ounces " by 12 are pounds.

APPLICATION.

1. Bring 672 ounces to pounds? Ans. 56 lbs.
2. Bring 1136 drams to ounces? " 142 ozs.
3. Bring 252 scruples to drams? " 84 dr.
4. Bring 320 grains to scruples? " 16 scr.
5. Bring 192 scruples to ounces? " 8 ozs.
6. Bring 80640 grains to pounds? " 14 lbs.
7. Bring 317 grains to scruples? Ans. 15 scr. 17 grs.
8. Bring 406 ounces to pounds? " 33 lbs. 10 ozs.
9. Bring 4020 grains to scruples? " 201 scr.
10. Bring 157 drams to ounces? " 19 ozs. 5 drs.

TROY WEIGHT.

CASE II.

Q. Recite the table referring to this case?

A. Grains divided by 24 are dwts.
Pennyweights " by 20 are ounces.
Ounces " by 12 are pounds.

APPLICATION.

1. Bring 1872 grains to pennyweights? Ans. 78 dwts.
2. Reduce 2320 dwts. to ounces? " 116 ozs.
3. Reduce 144000 grains to pounds? " 25 lbs.
4. Reduce 595 pennyweights to a higher denomination? Ans. 29 ozs. 15 dwts.
5. Reduce 175 dwts. to ounces? " 8 ozs. 15 "
6. Reduce 94 ounces to pounds? " 7 lbs. 10 ozs.
7. Reduce 1721 grains to dwts.? " 71 dwts. 17 grs.
8. Reduce 86400 grains to lbs.? " 15 lbs
9. Reduce 5749 dwts. to a higher denomination? Ans. 287 ozs. 9 dwts.
10. In 2385 grains, how many spoons, each weighing 6 dwts. 15 grains? Ans. 15
11. In 160632 grains of pure gold, how many lbs, ozs., and dwts.? Ans. 27 lbs. 10 oz. 13 dwts.

LONG MEASURE.

CASE II.

Q. Recite the second form of the table?

A. Barley corns divided by 3 are inches.
Inches " by 12 are feet.
Feet " by 3 are yards.
Yards " by 5½ are perches.
Perches or poles " by 40 are furlongs.
Furlongs " by 8 are miles.
Miles " by 3 are leagues.
Also, miles " by 60 are geographic degrees.

APPLICATION.

1. In 1837440 inches, how many miles? Ans. 29m.
2. In 19753 yards, how many furlongs?
   Ans. 89 fur. 173 yds.
3. In 590057 inches, how many leagues?
   Ans. 3 lea. 2 fur. 110 yds. 1 ft. 5 in.
4. In 22800 barley corns, how many miles?
   Ans. 11 m. 7 ft. 38 p. 2 yds. 2 ft.
5. In 34594560 inches, how many miles? Ans. 546m.
5. In 720 perches, how many furlongs? " 18 fur.
7. In 144 feet, how many yards? " 48 yds.
8. In 1140480 inches, how many leagues? " 6 lea.
9. In 35200 yards, how many miles? " 20 m.

CLOTH MEASURE.

CASE II.

Q. Recite the table corresponding to this case?
A. Nails divided by 4 are quarters.
   Quarters " by 4 are yards.
Q. Bring nails to English Ells?
A. Nails divided by 4 are quarters.
   Quarters " by 5 are English Ells.
Q. Bring nails to Flemish Ells?
A. Nails divided by 4 are quarters.
   Quarters " by 3 are Flemish Ells.
Q. Bring nails to French Ells?
A. Nails divided by 4 are quarters.
   Quarters " by 6 are French Ells.

APPLICATION.

1. Reduce 36 quarters to yards? Ans. 9 yds.
2. Reduce 180 quarters to Flemish Ells? " 60 E. F.
3. Reduce 95 quarters to English Ells? " 19 E. E.
4. Reduce 126 quarters to French Ells? " 21 E. Fr.
5. Reduce 318 nails to a higher denomination?
   Ans. 19 yds. 3 qr. 2 na.
6. Reduce 2528 nails to yards? Ans. 158 yds.

LIQUID MEASURE.

CASE II.

Q. Recite the table agreeing with case second?

A. Pints divided by 2 are quarts.
Quarts " by 4 are gallons.
Gallons " by 63 are hogsheads.
Hhds. " by 4 are tuns.

APPLICATION.

1. How many quarts in 34 pints? Ans. 17 qts.
2. How many gallons in 100 quarts? " 25 gals.
3. How many hhds. in 315 gallons? " 5 hhds.
4. How many gallons in 800 pints? " 800 gals.
5. How many tuns in 12096 pints? " 6 tuns.
6. How many hogsheads in 1930 pints?
Ans. 7 hhds. 41 gals. 2 qts.
7. In 10707 pints, how many tuns, hogsheads, gallons, quarts and pints? Ans. 5 ts. 1 hhd. 15 gals. 1 qt. 1 pt.

PROMISCUOUS EXAMPLES.

1. In 5 hhds. of wine, how many pints?
2. In 27 hhds. 16 gallons, how many quarts?
3. In 942 tuns, how many pints?
4. In 50400 pints, how many tuns?
5. In 3240 half pints, how many hogsheads, gallons, and quarts?

TIME.

The denominations are seconds, minutes, hours, days, weeks, months, and years.

CASE I.—*To bring years to seconds.*

Years multiplied by 12 are months.
Months " by 4 are weeks.
Weeks " by 7 are days.
Days " by 24 are hours.
Hours " by 60 are minutes.
Minutes " by 60 are seconds.

APPLICATION.

1. Bring 1 year, 10 months, 2 weeks, 3 days, 12 hours 10 min. 15 sec. to seconds? Ans. 31407015 sec.
2. Bring 30 minutes to seconds? " 1800
3. Bring 12 years to months? " 144
4. Bring 3 days, 5 hours, 29 minutes to minutes? Ans. 4649 min.
5. Bring 37 weeks, 5 days to minutes? " 380160

CASE II.

Q. Recite the table corresponding to this case?

A. Seconds divided by 60 are minutes.
Minutes " by 60 are hours.
Hours " by 24 are days
Days " by 7 are weeks.
Weeks " by 4 are months.
Months " by 12 are years.

LAND OR SQUARE MEASURE.

CASE II.

APPLICATION.

1. Reduce 160 roods to acres? Ans. 40 acres.
2. Reduce 487 sq. yds. to sq. perches? " 28 sq. pr.
3. Reduce 878 sq. ft. to square yards? " 42 sq. yds.
4. Reduce 600 perches to acres? " 3 a. 3 rds.
5. In 5600 perches, how many acres? " 35 acres.
6. In 4850 perches, how many acres? " 30a. 1r. 10p
7. In 484 sq. yds. how many sq. pr.? " 16
8. In 5684580 sq. inches, how many roods and perches? Ans. 3rds. 25 perches.
9. In 576 sq. inches, how many sq. feet? Ans. 4
10. In 115520 sq. inches, how many sq. yds.? " 120
11. In 37974 square inches, how many square yards, square feet, and square inches. Ans. 29 sq. yds. 2 sq. ft. 102 sq. in.
12. In 3946 perches, how many acres, roods, and perches? Ans. 24 a. 3 rds. 26 ps.

DRY MEASURE.

The denominations are 2 pints make 1 quart, 8 quarts one peck, and 4 pecks one bushel.

Reduction Descending.

Q. How do you bring bushels to pints?

RULE 1.—Bushels multiplied by 4 are pecks.
Pecks " by 8 are quarts.
Quarts " by 2 are pints.

1. In 4 bushels, how many pints?

4
—
16 pecks.
8
——
128 quarts.
2
——
256 pints.

RULE 2.—Pints divided by 2 are quarts; quarts divided by 8 are pecks; pecks divided by 4 are bushels.

APPLICATION.

1. Bring 32 pecks to quarts? Ans. 256 qts.
2. Reduce 7 bushels to pecks? " 28 p.
3. Bring 12 bushels to pints? " 768 pts.
4. Bring 15 bushels, 3 pecks, to pecks? " 63 p.
5. Bring 25 bushels, 1 peck, 2 quarts, 1 pint, to pints? Ans. 1621 pts.

MOTION OR CIRCLE MEASURE.

The denominations are—

60 seconds	make	1 minute,	'
60 minutes	"	1 degree,	°
30 degrees	"	1 sign,	sig.
12 signs	"	1 revolution or circle.	

The 12 signs of the Zodiac are as follows:

Aries.	*Leo.*	*Saggitarius.*
Taurus.	*Virgo.*	*Capricornus.*
Gemini.	*Libra.*	*Aquarius.*
Cancer.	*Scorpio.*	*Pisces.*

APPLICATION.

1. Reduce 24° to minutes? Ans. 1440
2. In 5 signs, how many seconds? Ans. 540000
3. Reduce 11 sig. 12° to degrees? " 342°
4. How many second are in 4 sig. 3° 18m. 27 seconds? Ans. 443907"

NAMES OF THE TWELVE CALENDAR MONTHS.

January, 1st month has 31 days.
February, 2d month has 28, (except leap year, which gives it 29.)
March, 3d month has 31 days.
April, 4th " 30 "
May, 5th " 31 "
June, 6th " 30 "
July, 7th " 31 "
August, 8th " 31 "
September, 9th " 30 "
October, 10th " 31 "
November, 11th " 30 "
December, 12th " 31 "

OTHERWISE EXPRESSED.

The *fourth, eleventh, ninth,* and *sixth,*
Have 30 days to each affixed,
And every other 31,
Except the second month (February) alone,
Which has but twenty-eight in fine,
Till leap year gives it twenty-nine.

Note.—Every fourth year is leap year.

Q. How many days in a common year?

A. Three hundred and sixty-five days, and six hours.

Q. How many days in a leap year?

A. Three hundred and sixty-six.

ILLUSTRATION.

365 days, 6 hours, in a common year.
366 " 0 in leap year.

So that $4 \times 6 = 24$ hours, or 1 day, gained every fourth year, which is called leap year.

QUESTIONS FOR EXERCISE IN REDUCTION.

1. In 40 dollars, how many cents? Ans. 4000.
2. In 96 furlongs, how many miles? " 12.
3. In 245 days, how many weeks? " 35.
4. In 6 tons, 15 cwt. how many cwts.? " 135.
5. In 96 half cents, how many cents? " 48.
6. In 2520 perches, how many roods? " 63.
7. In 176 pecks, how many bushels? " 44.
8. In 108 pints, how many quarts? " 54.
9. In 720 seconds, how many minutes? " 12.
10. In 474 gals. how many hhds? Ans. 7 hhd. 33 gals.
11. In 103 pints, how many quarts? " 51 qts. 1 pt.
12. In 273 miles, how many leagues? Ans. 91.
13. In 108 dwts. how many ounces? Ans. 5 oz. 8 dwts.
14. In 250 shillings, how many Ls? " 12 Ls. 10 s.
15. In 21 scruples, how many drams? Ans. 63.
16. In 203 days, how many weeks? " 29.
17. In 64 nails, how many yards? " 4.
18. In 84 drams, Avoirdupois weight, how many ounces? Ans. 5 ozs. 4 drs.
19. In 4320 minutes, how many days? Ans. 3.
20. In 400 cwt. how many tons? " 20.
21. In 32 furlongs, how many miles? " 4.
22. In 12960 grs. of gold, how many lbs? Ans. 2lbs. 3oz.
23. In 13 lbs. Avoirdupois, how many drs? Ans. 3328.
24. In 1 ton, how many pounds? " 2240.
25. What will 1 ton of iron come to at 6¼ cts. per lb.? Ans. $140.00.

26. In 27 lbs. 6 ozs. gold, how many grs? Ans. 158400.
27. In 252 bushels, how many quarts? " 8064.
28. In 8064 quarts, how many bushels? " 252.
29. In 4840 sq. yds. how many sq. per.? " 160.
30. In 156 qrs. how many Ells Flemish? " 52.
31. In 760 " how many Ells English? " 152.
32. In 840 " how many Ells French? " 168.
33. In 27552 lbs. how many cwts.? " 984.
34. In 584621 gallons, how many tons?
Ans. 9279 tons, 3 hhds. 44 gals.
35. In 86400 seconds, how many days? Ans. 1.
36. Bring 1 mile to barley corns? Ans. 190080.
37. Bring 1 ton to drams? " 573440.
38. Bring 5 lbs. Troy weight to grains? " 28800.
39. Bring 5 lbs. Avoirdupois wt. to drs.? " 1280.
40. Bring 5 lbs. Apothecaries wt to grs.? " 28800.
41. Bring 5 signs of the Zodiac to sec.? " 540000.
42. Bring 3 ds. 5 hrs. 29 m. to minutes? " 4649.
43. Bring 37 yards, 1 foot, to feet? " 112.
44. Bring 880 pints to gallons? " 110.
45. In 1 hhd. of ale, how many pints? " 504.
46. In 2310 pints, how many bushels?
Ans. 36 bushs. 0 p. 3 qts.
47. In 4032 qts. of bush. how many hhds? Ans. 16.
48. In 36 bu. 3 qts. of corn, how many pts.? " 2310.
49. In 6888 hours, how many weeks? Ans. 41.
50. In 41 weeks, how many hours? " 6888.
51. In 2850 perches, how many acres?
Ans. 17 acres, 3 roods, 10 perches.
52. In 17a. 3r. 10p. how many perches? Ans. 2850.
53. In 72 leagues, how many yards? Ans. 380160.
54. In 3 tons of lead, how many lbs.? " 6720.
55. In 7 miles, how many feet? " 36960.
56. In 484 yards, how many perches? " 88.
57. In 2400 English sovereigns, how many dollars? (allowing the value of a 'sovereign' to be $4.80.)
Ans. $11520.

58. In 70 boxes of sugar, each 12 lbs. how many cwts.?
Ans. 7 cwt. 2 qrs.

59. In 6272640 sq. in. how many acres? Ans. 1.

60. In 1000 dimes, how many Eagles? " 10.

61. In 423 barley corns, how many inches? " 141.

62. In 820 cwts., how many tons? " 41.

63. In 100 dollars, how many half dimes? Ans. 2000.

64. In 5280 feet, how many miles? " 1.

65. How many times does a regular clock strike in a days? Ans. 156.

66. How many seconds in a year, allowing it to be 365 days, 6 hours? Ans. 31557600.

CUBIC OR SOLID MEASURE.

REPEAT THE TABLE.

1728 solid or cubic inches make 1 solid foot.

40 feet of round timber, or 50 feet of hewn timber, } make 1 ton.

ALSO,

27 solid or cubic feet make 1 yard.

128 solid feet, or 8 feet long, 4 feet wide, and 4 ft. high, } make 1 cord of wood.

1. If a pile of wood be 40 feet long, 4 feet wide, and 4 feet high, how many cords are therein? Ans. 5 cords.

2. Suppose a pile of wood is 48 feet long, 5 feet wide, and 3½ feet high, how many cords in it? Ans. $6\frac{9}{16}$ cds.

Q. How is the operation performed?

A. By multiplying the length, width, and height together, and dividing the product by 128 for the number of cords.

The cellar of a house is 20 feet long, 30 wide, heighth or depth 10 feet, what will the digging thereof come to at 7 cts. per cubic yard? Ans. $15.55\frac{5}{9}$.

Q. How is the above question done?

A. By multiplying the length, breadth, and depth together, and dividing by 27 for the number of cubic yards we get $222\frac{2}{3}$ cubic yds. at 7c. per is $15.55\frac{5}{9}$.

Q. What is a cubic foot?

A. It is 12 inches long, 12 inches broad, and 12 inches deep. $12 \times 12 + 12 = 1728$ solid inches.

Q What is a cubic yard?

A. In length it is 3 feet, in breadth 3 feet, and in depth 3 feet; consequently, $3 \times 3 \times 3 = 27$ feet in a solid yard, $3^3 = 27$.

Q. How many feet in a square yard?

A. Nine feet, $3 \times 3 = 9$ feet in a square yard.

Q. How is a number squared?

A. By multiplying the number by itself, as $4^2 = 16$, or $4 \times 4 = 16$.

Q. How is the number cubed?

A. By multiplying the square of a number by the same number, as $4 \times 4 = 16 \times 4 = 64$, or $4 \times 4 \times 4 = 16$.

What is the square of 5? What is the cube of 5?
What is the square of 8? What is the cube of 8?
What is the square of 9? What is the cube of 9?

1. How many solid inches in 2 solid feet? Ans. 3456.
2. How many solid ft. in 345600 solid in.? " 200 ft.
3. How many solid ft. in 691200 solid in.? " 400 ft.
4. In 2 cords of wood, how many solid ft? " 256 ft.
5. In 30 cords, how many feet? Ans. 3840 ft.
6. A floor in a certain building is 22 feet long, and 18 feet wide, how many feet of boards will cover the floor?

Ans. $22 \times 18 = 396$ ft.

7. A board is 20 feet, 9 inches long, and 14 inches wide, how many feet are contained therein?

Ans. 24 ft. $2\frac{1}{2}$ in.

A SYNOPSIS OF THE PRECEDING RULES OF ARITHMETIC.

☞ The learner should be required to recite the foregoing tables and rules in Reduction, ascending and descending.

FEDERAL MONEY.

Add	6	07	8
"	3	09	7
"	7	06	3
"	9	03	2
	25	27	0

Q. Why do you prefix a cipher before 7 cents in the first line.

A. I must always do so, when the cents are less than 10: thus, 7, 9, 6, and 3, in the column of cents are less than 10, consequently, ciphers must be prefixed thus: 01 for 1 cent, 02 for 2 cents, 03 for 3 cents, 04 for 4 cents, 05 for 5 cents, 06 for 6 cents, 07 for 7 cents, and so on up to 10.

If I buy 4 lbs. of coffee for 50 cents, 3 lbs. of tea for $2.50, 7 lbs. candles for 87½ cents, and 1 gallon of wine, for $1.93¾ cents, what must I pay for them?

Coffee, - - - -	$0.50
Tea, - - -	2.50
Candles, - - -	0.87½
Wine, - - -	1.93¾

My cook has bought in market a turkey for $1.87½ cts. a pair of ducks for $1.18¾, a quarter of lamb for 43¾ cents, a quarter of veal for $1.37½, a piece of beef for 68¾ cents, and a peck of apples for 12½ cents, what sum must I give to pay for the articles?

A turkey, - - -	$1.87½
Pair of ducks, - -	1.18¾
Quarter of lamb, - -	43¾
Quarter of veal, - -	1.37½
Beef, - - . -	68¾
Apples, - - -	12½

ADDITION AND SUBTRACTION OF FEDERAL MONEY.

1. If from $100.00, there be paid at one time $17.28¼, at another time $10·00½, and another time $37.15, how much will remain? Ans. $35.56.

2. Subtract ½ cent from $100.

3. Subtract ¾ of a cent from $100.

4. Subtract 87½ cents from $50.06¼.

5. Subtract 7 cents from $20.

6. Subtract 1¼ cents from $20.

7. Subtract 9¾ cents from $104.06¼.

8. Take ¼ of a cent from $100.

ENGLISH OR STERLING MONEY.

The denominations are farthings, pence, shillings, and pounds.

4 farthings make	- - -	1 penny,	*d.*
12 pence "	- - -	1 shilling,	*s.*
20 shillings "	- - -	1 pound,	*L.*

This standard is said to have been fixed in the reign of Richard 1st, by persons from the Eastern parts of Germany, called *Easterlings*, and hence, the word sterling, which is now applied to all lawful money of Great Britain.

	L.	*s.*	*d.*		*L.*	*s.*	*d.*
Example 1.	2	3	4	Example 2.	7	2	4½
	7	1	2		13	7	6¾
	9	7	3		4	5	2
	5	2	2½		10	18	10¾
	£23	13	11¼.		£35	14	0

Rule.—Add up the farthings, divide the sum by 4, and if there be a remainder, set down the remainder as so many farthings, carry the quotient to the pence column, and add them up as in common addition, and divide the sum by 12, because 12 pence is a shilling, if there be a remainder set it down under the pence column, and carry the quotient to the column of shillings. In like manner add the shillings, and divide the sum by 20, for

pounds, because 20*s.* is a pound; if there be a remainder set it down under the column for shillings, and carry the quotient to the pounds, the pounds added up above 10 as in common addition, will give the result required.

REDUCTION DESCENDING.

RULE 1.—Pounds multiplied by 20 are shillings, shillings multiplied by 12 are pence, pence multiplied by 4, are farthings.

	L.	*s.*	*d.*	
Bring	10	17	6¾	to farthings.
	20			
	217			
	12			
	2610			
	4			
	10443	farthings.		

	L.	*s.*	*d.*	
Bring	68	16	6½	to farthings.
Bring	17	15	3¼	to farthings.
Bring	25	10	8¼	to farthings.
Bring	100	19	9¾	to farthings.
Bring	19	19	11¾	to farthings.

Q. What are farthings added above? Ans. 4.
Q. What are pence added above? " 12.
Q. What are shillings added above? " 20.
Q. What are pounds added above? " 10.

Q. How do you bring pounds, shillings, pence and farthings to farthings?

A. The pounds I multiply by 20, by Rule 1st, and add in the shillings, (if any,) the shillings I multiply by 12, to bring them to pence, the pence I multiply by 4, to bring them to farthings.

REDUCTION ASCENDING.

Q. How is this kind of Reduction performed?

A. By Division.

Q. What is the rule, Rule 2d.

A. Farthings divided by 4 are pence, *d.*
Pence " by 12 are shillings, *s.*
Shillings, " by 20 are pounds, *L.*

EXAMPLE.—Bring 30096 farthings to pounds.
Bring 96000 farthings to pounds.
Bring 19199 farthings to pounds.

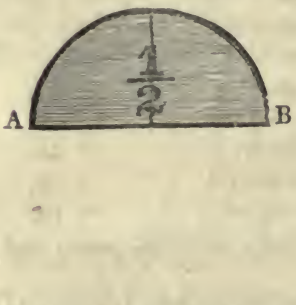
PROMISCUOUS EXAMPLES IN STERLING MONEY.

	L.	*s.*	*d.*
Add	69483	19	8¼
"	39678	15	4¼
"	84374	15	3¾
"	94287	17	4¼
"	93687	16	9¾

	L.	*s.*	*d.*
Add	648	17	6
"	365	14	9
"	487	17	8¼
"	935	15	9¾
"	787	17	6¾

SUBTRACTION OF STERLING MONEY.

From	1008	19	6¾
Take	784	18	7¼
Rem'r.	224	00	11½



Q. How is this rule performed?

RULE.—As 4 farthings make 1 penny, 4 will be a common denominator, and in the above example, where 7 pence is the *subtrahend*, and 6 pence the *minuend*, you take 7 from 12, and the remainder is 5, which added to 6 in the minuend make 11 pence, which set down as the

difference in the pence column, then carry 1, because you speak of 12 to the shillings, 1 carried to 18, make 19; 19 from 19 leaves 0: then subtract the pounds, as in common subtraction.

From	1998	17	$6\frac{3}{4}$	From	198	17	$8\frac{1}{4}$
Take	1348	15	$9\frac{1}{4}$	Take	139	19	$11\frac{3}{4}$

I have a purse of money containing £100 2*s*. $4\frac{1}{4}$*s*, if I take out £60 7*s*. $8\frac{3}{4}$*d*. what sum will be left?

Ans. £939 14*s*. $7\frac{3}{4}$*d*.

FEDERAL MONEY.

Lent a man $400, he now returns 211.12\frac{1}{2}$ cts. how much is unpaid? Ans. 188.87\frac{1}{2}$.

My carpenter's bill rendered this day is 110.95\frac{3}{4}$ cts. I paid him 90.10\frac{3}{4}$, how much is the balance due him?

Ans 20.84\frac{1}{4}$.

ADDITION OF COMPOUND NUMBERS.

Q. What is the object of adding compound numbers?

A. To unite parts of the same denomination in such a manner that their units may stand under each other.

Q. Where do you begin to add?

A. At the right hand column.

Q. Why so?

A. Because the addition of compound numbers depends on the same principles, as, that of simple numbers.

Q. How do you proceed?

A. By adding separately the sum of each column, always recollecting how many parts of each denomination it takes to make one of the next higher.

Q. By what do you divide the amount?

A. By as many of that denomination as will make one of the next greater, as before.

Q. After dividing, it there should be a remainder, how do you proceed?

A. The remainder is set down, and the quotient pro-

duced by dividing, carried to the next highest denomination. In this manner, the process is continued until all the denominations have been added up.

TROY WEIGHT.

	lbs.	*ozs.*	*dwts.*	*grs.*
Add	17	3	15	11
"	13	2	13	13
"	15	3	14	14
"	13	10	0	0
"	12	1	0	17
"	0	0	13	14
	71	9	17	21

	lbs.	*ozs.*	*dwts.*	*grs.*
Add	17	3	15	17
"	14	2	17	13
"	19	7	15	19
"	18	6	17	13
"	18	5	18	22

AVOIRDUPOIS WEIGHT.

	cwts.	*qrs.*	*lbs.*	*ozs.*	*drs.*
Add	15	2	15	15	15
"	2	3	3	14	13
"	12	2	13	14	14
"	10	1	17	15	0
"	12	1	10	0	10
"	13	2	17	12	12
	67	1	23	10	0

	tons.	*cwts.*	*qrs.*	*lbs.*	*ozs.*	*dr.*
Add	2	17	3	13	8	7
"	1	16	0	10	0	6
"	5	14	2	17	6	8
"	9	11	3	18	5	13
"	14	12	1	14	10	14

APOTHECARIES WEIGHT.

	lbs.	*ozs.*	*drs.*	*scr.*	*grs.*
Add	12	10	5	2	18
"	8	4	7	1	14
"	9	8	7	2	10

	lbs.	*ozs.*	*drs.*	*scr.*	*grs.*
Add	8	9	4	1	19
"	10	11	7	2	14
"	14	8	5	2	15

LONG MEASURE.

	lea.	*ms.*	*fur.*	*pr.*
Add	5	2	4	17
"	16	1	3	10
"	72	0	5	24
"	526	0	3	12
"	834	2	6	34
"	38	0	3	12
	1493	2	2	29

	yds.	*fl.*	*in.*
Add	16	2	11
"	1	1	9
"	14	2	11
"	99	1	8
"	108	2	10
"	436	2	7

CLOTH MEASURE.

	yds.	*qrs.*	*n.*
Add	75	3	2
"	163	1	3
"	245	2	0
"	738	3	1
"	1786	2	3
	3009	1	1

	E. E.	*qrs.*	*n.*
Add	72	3	2
"	98	2	3
"	47	1	2
"	96	2	2
"	149	1	3

LAND MEASURE.

	acs.	*rds.*	*pr.*
Add	39	2	37
"	62	1	17
"	68	0	38
"	129	3	12
"	532	1	18
	832	2	2

	acs.	*rds.*	*pr.*
Add	4968	3	27
"	9484	2	32
"	9694	1	26
"	4947	3	34

LIQUID MEASURE.

	tons.	*hhds.*	*gals.*
Add	18	2	54
"	63	1	39
"	327	0	04
"	46	1	19
"	285	3	28
	741	1	18

	hhds.	*gals.*	*qts.*	*pts.*
Add	346	42	3	1
"	27	36	1	0
"	468	24	1	0
"	543	37	2	1
"	964	36	3	1

DRY MEASURE.

	bush.	*pks.*	*qts.*
Add	37	2	1
"	182	3	2
"	422	1	0
"	162	3	1
"	357	0	2
	1163	1	6

	bush.	*pks.*	*qts.*	*pts.*
Add	42	1	5	1
"	63	2	3	0
"	44	3	7	0
"	65	2	6	1

TIME.

	yrs.	*ms.*	*ws.*	*ds.*	*hrs.*
Add	17	11	3	5	20
"	172	9	2	3	17
"	35	7	3	6	22
"	4	10	0	4	16
"	0	6	0	3	19
	231	6	3	3	22

	hrs.	*ms.*	*sec.*
Add	20	52	40
"	122	12	35
"	68	9	17
"	135	17	12
"	24	35	28

MOTION OR CIRCLE MEASURE.

	sign,	*dgs.*	*mins.*	*secs.*
Add	1	5	37	42
"	1	7	26	12
"	4	8	26	11
"	1	4	32	17
"	3	6	0	47
	11	2	3	9

	sign,	*degs.*	*m.*	*secs.*
Add	1	4	37	42
"	5	4	44	48
"	9	12	48	40
"	9	14	42	48

MULTIPLICATION OF COMPOUND NUMBERS.

Q. What is the utility of compound multiplication?

A. To find the product of numbers, one of which is compound, and the other, a simple factor.

RULE.

Multiply the compound quantity by the simple multiplier, beginning with the lowest denomination of the multiplicand; this done, divide the product by the number which it takes to make one of the next superior, putting down the remainder, (if any,) we then add the quotient last produced to the product of the next denomination by the multiplier to reduce this sum, putting down the remainder as before, and proceed in this manner through all the denominations to the last, which is to be multiplied like a simple number.

ILLUSTRATION OF THE RULE.

Suppose we multiply 4 cwt. 2 qrs. 16 lbs. by 9, we begin with the lowest denomination, and multiply 16 lbs. by

9, and find the product is 144 lbs. we divide 144 lbs. by 28, because it takes 28 lbs. to make 1 quarter, which is one of the next superior, and find the quotient is 5, and a remainder of 4, which we set down. We next multiply the next highest number 2 quarters by 9 'adding in' the former quotient, and we get 23 quarters; we reduce this sum by dividing it by 4, because 4 quarters make 1 cwt. and set down the remainder 3, which is 3 quarters, and proceed in the same manner, always observing that the last number is to be multiplied like a simple number.

Note.—When the multiplier exceeds 12, that is, when it is so large that it may be inconvenient to multiply by the whole at once. The shortest method is to resolve it into factors, if it can be done—if not, into powers of 10, or thus:

FEDERAL MONEY.

Multiply \$104 33 3
9

\$938 99 7 same as decimals.

or thus:

Multiply \$104 33 3
10

1043 33 0
104 33 3 By adding a cipher minus the given number as before.

\$938 99 7

ENGLISH MONEY.

	L.	*s.*	*d.*					
Multiply	12	3	6¾	by 8?	Multiply	9	11	4¼ by 5?

AVOIRDUPOIS WEIGHT.

	cwt.	*qrs.*	*lbs.*			*cwt.*	*qrs.*	*lbs.*	
Multiply	1	2	6	by 70?	Multiply	1	3	14	by 21?

TROY WEIGHT.

	lbs.	*ozs.*	*dwts.*	*gr.*			*lbs.*	*ozs.*	*dwts.*	*grs.*	
Multiply	67	5	16	1	by 2?	Multiply	17	9	14	2	by 10?

12

APOTHECARIES WEIGHT.

	lbs.	*ozs.*	*drs.*	*scr.*	*grs.*	
Multiply -	12	10	5	2	18	by 5?
"	16	6	6	2	18	by 4?

LONG MEASURE.

	lea.	*m.*	*fur.*	*pr.*			*yds.*	*ft.*	*in.*	*b. c.*	
Multiply	5	2	4	17	by 5?	Mul.	16	2	11	2	by 6?

CLOTH MEASURE.

	yds.	*qrs.*	*ns.*			*yds.*	*qrs.*	*ns.*	
Multiply	75	3	2	by 12?	Multiply	35	2	2	by 15?

TIME.

	yrs.	*m.*	*w.*	*d.*	*h.*	
Multiply	17	11	3	5	20	by 8?
"	0	6	0	3	19	by 5?

LAND MEASURE.

	acs.	*rds.*	*pr.*			*acs.*	*rds.*	*pr.*	
Multiply	39	2	25	by 8?	Multiply	45	3	32	by 5?

LIQUID MEASURE.

	tuns.	*hhds.*	*gals.*			*tuns.*	*hhds.*	*gals.*	
Multiply	18	2	54	by 10?	Multiply	6	1	23	by 8?

DRY MEASURE.

	bush.	*pks.*	*qts.*			*bush.*	*pks.*	*qts.*	
Multiply	37	2	1	by 8?	Multiply	44	1	5	by 10?

MOTION OR CIRCLE MEASURE.

	sign,	*deg.*	*ms.*	*sec.*			*sign,*	*deg.*	*ms.*	*sec.*	
Multiply	1	5	37	42	by 5?	Mul.	3	8	22	25	by 10?

SUBTRACTION OF COMPOUND NUMBERS.

This operation is performed in the same way as the subtraction of whole numbers, except with regard to the number, which it is necessary 'to borrow' from the higher denominations, in order to perform the partial subtractions, when the lower number exceeds the upper.

AVOIRDUPOIS WEIGHT.

	cwts.	*qrs.*	*ozs.*	*lbs.*	*drs.*		*cwts.*	*qrs.*	*ozs.*	*lbs.*	*drs.*
From	22	1	7	6	13	From	5	0	17	5	9
Take	13	0	8	8	14	Take	8	3	21	1	7
	9	0	26	13	15						

TROY WEIGHT.

	lbs.	*ozs.*	*dwts.*	*grs.*
From	17	9	14	2
Take	14	5	14	18

	lbs.	*ozs.*	*dwts.*	*grs.*
From	7	3	14	11
Take	3	7	15	20

APOTHECARIES WEIGHT.

	lbs.	*ozs.*	*drs.*	*scr.*	*grs.*
From	12	10	5	2	18
Take	6	9	6	2	18

	lbs.	*ozs.*	*drs.*	*scr.*	*grs.*
From	8	4	6	2	16
Take	4	8	7	1	19

LONG MEASURE.

	lea.	*ms.*	*fur.*	*pr.*
From	9	2	7	17
Take	6	1	6	25

	m.	*fur.*	*pr.*	*yds.*
From	10	5	20	4
Take	8	7	35	5

CLOTH MEASURE.

	yds.	*qrs.*	*n.*
From	75	3	2
Take	35	2	3

	E. E.	*qrs.*	*n.*
From	75	4	3
Take	63	4	3

TIME.

	yrs.	*ms.*	*ws.*	*ds.*	*hrs.*
From	17	6	3	6	20
Take	12	8	3	5	18
	4	10	0	1	2

	hrs.	*ms.*	*sec.*
From	20	52	62
Take	12	58	58

LAND MEASURE.

	acs.	*rds.*	*pr.*
From	45	2	33
Take	39	3	35
	5	2	38

	acs.	*rds.*	*pr.*
From	24	0	05
Take	22	3	35

LIQUID MEASURE.

	tuns.	*hhds.*	*gals.*	*qts.*	*pt.*
From	18	2	52	2	1
Take	12	3	58	3	0
	5	3	56	3	1

	hhds.	*gals.*	*qts.*	*pt.*
From	6	42	3	1
Take	3	33	3	1

DRY MEASURE.

	bush.	*pks.*	*qts.*
From	44	1	5
Take	37	2	6
	6	2	7

	bush.	*pks.*	*qts.*	*pt.*
From	3	3	3	1
Take	2	3	7	1

MOTION OR CIRCLE MEASURE.

	sign,	*dgs.*	*mins.*	*secs.*
From	3	8	22	35
Take	1	5	37	42

	degs.	*ms.*	*secs.*
From	37	33	35
Take	22	35	55

DIVISION OF COMPOUND NUMBERS.

A compound number may be divided by a simple number, by regarding each of the terms of the former, as forming a distinct dividend.

RULE.

Divide the highest term of the compound number by the given divisor, reduce the remainder, (if any) to the next lower denomination, adding it to the number of this denomination, and divide the sum by the divisor, reducing the remainder as before, and proceed, in this way through all the denominations to the last.

TROY WEIGHT.

ILLUSTRATION, 67 5 16 22 by 2.

Dividing by 2 is taking ½)33 8 18 11

Here 33 is the quotient, and 1 of a remainder, which is 1 lb. this reduced to the next lower denomination which is ounces, we get 12 + 5 = 17 ounces, this divided by 2, gives 8 in the quotient, and 1 of a remainder, which is 1 oz., this reduced to the next lower, which is pennyweights, and we get 20 + 16 = 36 pennyweights; 36 divided by 2 gives 18, and no remainder; the next lower denomination is 22 grains, and this also divided by 2 gives 11. All other compound quantities are divided upon the same principle.

LAND MEASURE.

	acs.	*rds.*	*pr.*					
Divide	45	3	32	by 8?	Divide	24	3	05 by 5?

CLOTH MEASURE.

	yds.	*qrs.*	*ns.*					
Divide	196	2	2	by 4?	Divide	48	3	3 by 6?

AVOIRDUPOIS WEIGHT.

	tons,	*cwt.*	*qrs.*	*lbs.*						
Divide	6	14	2	18	by 4?	Divide	8	12	3	24 by 8?

LIQUID MEASTRE.

	tuns,	*hhds.*	*gals.*	*qts.*						
Divide	6	4	44	3	by 3?	Divide	0	36	62	2 by 4?

DRY MEASURE.

	bush.	*pks.*	*qts.*					
Divide	64	2	6	by 4?	Divide	84	3	7 by 6?

MOTION OR CIRCLE MEASURE.

The moon revolves through 12 signs of the Zodiac in 27 days, 7 hours, 42 minutes, 48 seconds. In what time does it describe one sign? Ans. 2*d.* 6*h.* 38*m.* 34*s.*

LECTURE VIII.

FRACTIONS SIMPLIFIED.

DEFINITIONS.

Q. What is a fraction?

A. A fraction is an expression of a part, or some parts of any thing considered as a whole.

Q. How is it denoted?

A. By two numbers, one placed below the other, with a line between them as $\frac{3}{4}$, written three-fourths.

Q. What is the number above the line called?

A. It is called a numerator, (as before mentioned) from the French, *numerateur*, which determines the number of parts, it also represents a remainder after division.

Q. What is the number below the line called?

A. It is called a denominator, from the Latin, *denomino*, because it denominates the number of parts.

Q. What are the numerator and denominator considered?

A. They are generally considered the terms of a fraction?

Q. How are fractions arranged?

A. Into four classes, viz: *proper*, *improper*, *compound*, and *mixed*.

Q. What is a proper fraction?

A. It is that, whose numerator is less than the denominator, as $\frac{7}{8}$ or $\frac{9}{10}$, &c.

Q. What is a complex fraction?

A. It is that, which has a fraction in its numerator or denominator, or in both of them: thus, $\frac{5\frac{1}{2}}{6}$, $\frac{8}{9\frac{1}{4}}$, $\frac{4\frac{3}{4}}{7}$.

Q. What is an improper fraction?

A. It is that whose numerator is greater than the denominator, as $\frac{12}{5}$ or $\frac{7}{4}$, &c.

Q. What is a compound fraction?

A. It is the fraction of a fraction, or several fractions connected by the proposition *of* between them, as the $\frac{1}{4}$ and $\frac{1}{2}$ *of* $\frac{1}{2}$ *of* a dollar.

Q. What is meant by a mixed number?

A. A whole number, and a fraction together, as $3\frac{1}{4}$, or $12\frac{1}{2}$, &c.

Q. Can a whole, or integer number be expressed like a fraction?

A. Yes, by writing 1 below it as a denominator, as 3 is $\frac{3}{1}$, 4 is $\frac{4}{1}$ or 25 is $\frac{25}{1}$. This is evident, because a unit is neither a multiplier or a divisor, that is, you neither increase or decrease the value of any thing by multiplying or dividing by 1, as $6 \times 1 = 6$, $\frac{6}{1} = 6$.

Q. How is the value of an improper fraction obtained?

A. By dividing the numerator by the denominator, as $\frac{12}{4} = 3$, $\frac{20}{5} = 4$.

Q. What is a reciprocal fraction?

A. It is a fraction inverted, as $\frac{4}{3}$ is the reciprocal of $\frac{3}{4}$.

RULES.

1. When the numerator is less than the denominator, the fraction is less than 1, as $\frac{1}{8}$ or $\frac{1}{4}$, &c.

2. When the numerator is equal to it, the fraction is equal to 1, as $\frac{5}{5}$ or $\frac{8}{8}$, &c.

3. When the numerator is greater than the denominator, the fraction is greater than 1, as $\frac{6}{5}$ or $\frac{9}{8}$, &c.

Q. Can you tell by inspection when a proper fraction may be less than another, as $\frac{3}{4}$, $\frac{3}{5}$, $\frac{7}{8}$ or $\frac{7}{9}$?

A. Certainly, $\frac{3}{4}$ is greater than $\frac{3}{5}$, because the numerator does not shew a division of so many parts, for according to the meaning attached to the words numerator and denominator, it is plain, that a fraction is increased by

increasing its numerator, without changing its denominator, and that a fraction is diminished by diminishing its numerator, without changing its denominator; also, that a fraction is diminished when its denominator is increased without changing its numerator.

Q. What is meant by the greatest common measure of two numbers?

A. The largest number that will divide them without a remainder, to the lowest terms.

Q. When is a fraction in its lowest terms?

A. When no number but a unit, will measure both its terms.

Q. What is a prime number?

A. A number which can only be divided by itself, or a unit, as 7, 9, 13, 19, 5, 23, &c., (as before mentioned in the work.)

Q. What is a composite number?

A. A number which is equal to the products of its factors, as $28 = 7 \times 4$, $96 = 6 \times 8 \times 2 = 96$.

Q. What is an abstract fraction?

A. An abstract fraction is a fraction derived from another, by means of Reduction.

PREPARATORY QUESTIONS.

CASE I.

I. To find the greatest common measure of two given numbers.

RULE.

"The greater by the less divide,
The less, by what remains beside;
The last divisor still again,
By what remains, till *nought* remains;
And what divides and leaveth *nought*,
Will be the common measure sought."

1. What is the common measure of 112 and 120?

Ans. 8.

2. What is the greatest common measure of 379 and 403? Ans. 31.

3. What is the least common measure of 56 and 108? Ans. 4.

4. What is the greatest common measure of 57 and 285? Ans. 57.

5. What is the greatest common measure of 169 and 175? Ans. 1.

6. What is the greatest common measure of 175 and 455? Ans. 35.

CASE II.

II. To find the least common multiple of any given number.

RULE.—If the given numbers are prime to each other, their continued product is their least common multiple.

"If *not* all prime to all beside,
See what will two or more divide:
Divide the two or more thereby,
The same upon the quotients try;
And if thou canst divide them do;
'Till nought will measure any two,
Then the last quotients multiplied,
And all the numbers which divide
Continually; the product got,
Will be the multiple that's sought."

1. What is the least common multiple of 3, 5, 8 and 10?

Operation, 5)3 5 8 10

2)3 1 8 2

3 1 4 1 last quotients.

3

12

2 divisor.

24

common multiple, 5

120 answer.

2. Find the least common multiple between 12, 25, 30, and 42? Ans. 2100.

3. Find the least common multiple of 12, 16, 20, and 30? Ans. 240.

4. What is the least common multiple of 25, 35, 60 and 72? Ans. 12600.

5. What is the least common multiple that will measure 3, 4, 8, and 12? Ans. 24.

6. What number is the least, that 7, 8, 16, and 28, will measure? Ans. 112.

CASE IV.

To reduce fractional parts of a dollar to cents.

RULE.—Multiply the numerator by 100 (because 100 cents is a dollar) and divide by the denominator.

1. Bring $\frac{5}{8}$ of a dollar to cents?

$$\begin{array}{r} 100 \\ 5 \\ \hline 8)500 \end{array}$$

62½ or 62½ cts. $\frac{4}{8}$ may be reduced to $\frac{1}{2}$ because 4 will divide the numerator and denominator without a remainder, thus: $4)\frac{4}{8}=\frac{1}{2}$ reduced to its lowest terms. How many cents in $\frac{1}{2}$ of a dollar? Ans. $.50.

2. Bring $\frac{7}{8}$ of a dollar to cents? Ans. $.87½.

3. Bring $\frac{3}{8}$ of a dollar to cents? Ans. $.37½.

4. Bring $\frac{1}{8}$ of a dollar to cents? Ans. $.12½.

5. Bring $\frac{6}{8}$ or $\frac{3}{4}$ " to cents? Ans. $.75.

6. What number of cents in $\frac{90}{100}$ of a dollar? Ans. $.90.

7. What number of cents in $\frac{6}{10}$ of a dollar? Ans. $.60.

8. What number of cents in $\frac{4}{5}$ of a dollar? Ans. $.80.

CASE V.

To reduce fractions to their lowest denominations, and also into cents.

1. Reduce $\$\frac{19}{100}$ to its lowest terms? Ans. $\$\frac{19}{100}$ or 19 $\frac{19}{100}$ cts.

2. Reduce $\$\frac{75}{100}$ to its lowest terms? Ans. $\$\frac{3}{4}$ or 75 cts.

3. Reduce $\$\frac{4}{8}$ to its lowest terms? Ans. $\$\frac{1}{2}$ or $85\frac{1}{2}$ cts.
4. Reduce $\$\frac{4}{8}\frac{2}{4}$ to its lowest terms? Ans. $\$\frac{1}{2}\frac{3}{4}$ or $56\frac{1}{8}$ cts.
5. Reduce $\$\frac{2}{4}\frac{0}{0}$ to its lowest terms? Ans. $\$\frac{2}{5}$ or 40 cts.
6. Reduce $\$\frac{1}{2}\frac{8}{8}$ to its lowest terms? Ans. $\$\frac{9}{10}$ or 90 cts.
7. Reduce $\$\frac{1}{2}\frac{1}{4}$ to its lowest terms? Ans. $\$\frac{9}{16}$ or $56\frac{1}{4}$ cts.
8. Reduce $\$\frac{3}{5}$ to its lowest terms? Ans. $\$\frac{3}{5}$ or 60 cts.
9. Reduce $\$\frac{1}{2}\frac{4}{0}$ to its lowest terms? Ans. $\$\frac{2}{5}$ or 80 cts.

RATIO OF FRACTIONAL PARTS OF A DOLLAR.

1. What is the ratio between $\frac{1}{4}$ and $\frac{1}{2}$? Ans. 2.

From the preliminary examples it is evident that 2 quarters are = to a half; therefore, the ratio is as 1 to 2 as required.

2. What is the ratio between $\frac{1}{4}$ and $\frac{3}{4}$? Ans. 3.
3. What is the ratio between $\frac{1}{4}$ and $\frac{3}{4}$? Ans. 3.
4. What is the ratio between $\frac{1}{8}$ and $\frac{3}{8}$? Ans. 3.
5. What is the ratio between $\frac{1}{8}$ and $\frac{7}{8}$? Ans. 7.
6. What is the ratio between $\frac{8}{10}$ and $\frac{1}{10}$? Ans. 5.

USEFUL THEOREMS IN FRACTIONS.

Note to Teachers.—The learner should be required to recite these theorems and to apply them practically.

THEOREM I.

To ADD or SUBTRACT fractions which have the same common denominator, the SUM or DIFFERENCE of their numerators must be taken, and the common denominator written under the result.

THEOREM II.

To reduce fractions to the same denominator, the two terms of each of them must be multiplied, by the denominator of the other.

THEOREM III.

A fraction can be multiplied in two ways; namely, by multiplying its numerator or dividing its denominator. Thus, multiply $\frac{1}{30}$ by 5 = $\frac{5}{30}$ which reduced is $\frac{1}{6}$ or $\frac{1}{30} \div 5$ (by dividing the denominator 30 by 5) = $\frac{1}{6}$.

THEOREM IV.

A fraction can be divided in two ways, by dividing its numerator or multiplying its denominator: thus, divide $\frac{4}{5}$ by 4 by dividing the numerator we get $\frac{1}{5}$ and by multiplying the denominator $\frac{4}{5} \times 4 = \frac{4}{20} = \frac{1}{5}$ which is exactly the same.

THEOREM V.

Multiplication alone, according as it is performed on the numerator or denominator, is sufficient for the multiplication and division of fractions; that is, when you multiply the numerator you INCREASE, and when you multiply the denominator you DECREASE.

CASE I.

By multiplying the numerator the fraction is } multiplied.
By dividing the denominator the fraction is }

CASE II.

By dividing the numerator the fraction is } divided.
By multiplying the denominator the fraction is }

THEOREM VI.

To multiply a whole number by a fraction.

RULE.—Multiply the number by the numerator and divide by the denominator: or divide the number by the denominator and multiply the quotient by the numerator.

EXAMPLE.—Multiply 20 by $\frac{3}{4}$ first, $20 \times 3 = \frac{60}{4} = 15$ $\frac{20}{4} = 5 \times 3 = 15$.

COROLLARY.

Every common divisor of two numbers must also divide the remainder resulting from the division of the greater of the two by the less.

ADDITION.

CASE VI.

When the numerators are alike and not more than a unit.

RULE.—Multiply the numerator and denominator of the fraction having the least denominator by the common measure of the fractions.

Add $\frac{1}{4}$ and $\frac{1}{8}$, here $\frac{1}{4} \times 2 = \frac{2}{8}$ and $\frac{1}{8}$ make $\frac{3}{8}$ Ans.
" $\frac{1}{5}$ and $\frac{1}{6}$
" $\frac{1}{3}$ and $\frac{2}{5}$
" $\frac{1}{4}$ and $\frac{5}{11}$

OR THUS:

Add $\frac{1}{4}$ and $\frac{1}{8}$, here 4 and 8 make $\frac{12}{4)32} = \frac{3}{8}$ Ans.
and $4 \times 8 = 4)32$
" $\frac{1}{5}$ and $\frac{1}{6}$
" $\frac{1}{3}$ and $\frac{2}{5}$.

RULE.—Add the denominators together for a new numerator, and multiply them together for a new denominator.

CASE VII.

When the numerators are alike and more than a unit.

RULE.—Add the denominators together, and multiply their sum by the common numerator, and the product will be a new numerator; also, the product of the denominators will be a common denominator.

Add $\frac{3}{4}$ and $\frac{3}{7}$, here 4 and 7 make 11, which multiply by the numerator 3, which is common to both.

FIRST METHOD.

Thus: $11 \times 3 = 33$
And $4 \times 7 = 28$ $\frac{}{} = 1\frac{5}{28}$ Ans.

SECOND METHOD.

$\frac{3}{4} \times 8 = \frac{3}{32}$
$\frac{3}{7} \times 7 = \frac{3}{56}$, here $\frac{3}{32}$ and $\frac{3}{56} = \frac{5}{56}$, as above.

THIRD METHOD.

$\frac{3}{4} \times \frac{3}{7}$ by multiplying the numerators alternately by the denominators. $3 \times 8 = 24$, $3 \times 7 = 21$, and $7 \times 8 = \frac{5}{56}$

Add $\frac{3}{5}$ and $\frac{4}{6}\frac{1}{1}$, - - - Ans. $1\frac{3}{6}\frac{1}{0}$.
Add $\frac{1}{4}$ and $\frac{2}{3}$, - - - Ans. $1\frac{5}{12}$.
Add $\frac{5}{6}$ and $\frac{7}{11}$, - - - Ans. $1\frac{1}{6}\frac{3}{6}$.

Add $\frac{1}{2}$ and $\frac{5}{14}$. Here the ratio between the denominators is as 1 to 7; therefore, $\frac{1}{2} \times 7 = \frac{7}{14}$ and $\frac{5}{14}$ make $\frac{12}{14}$ or $\frac{6}{7}$ answer.

Add $\frac{1}{3}$ and $\frac{4}{15}$, - - - Ans. $\frac{7}{15}$.
Add $\frac{3}{4}$ and $\frac{9}{16}$, - - - Ans. $1\frac{5}{8}$.
Add $\frac{5}{8}$ and $\frac{5}{24}$, - - - Ans. $\frac{7}{12}$.
Add $\frac{5}{7}$ and $\frac{5}{21}$, - - - Ans. $\frac{10}{21}$.

CASE VIII.

To add mixed fractions.

RULE.—Find a common denominator by reducing the fractions to the lowest terms.

Add $17\frac{3}{4}$	$4)\frac{3}{4}$	$\frac{4}{5}$	$\frac{8}{8}$	$\frac{4}{20}$	$\frac{10}{20}$
" $19\frac{4}{5}$					
" $13\frac{8}{8}$	5)1	5	2	5	5
" $18\frac{4}{20}$					
" $19\frac{10}{20}$	2)1	1	2	1	1
	1	1	1	1	1
$89\frac{3}{20}$					

Multiply all the divisors together $4 \times 5 \times 2 = 40$ common denominator.

CASE IX.

To add mixed fractions whose numerators and denominators are unlike.

Add $15\frac{3}{4}$
Add $19\frac{4}{5}$

$35\frac{7}{8}$ $(\frac{7}{8})$

The operation can be performed thus, by cross multiplication $\frac{3}{4} + \frac{4}{5}$, $24 + 20 = \frac{44}{40} = 1\frac{7}{8}$ reduced, from whence the following Rule is deduced: multiply each numerator by all the denominators, except its own, for a new numerator, and all the denominators together for a new denominator.

Example.—Add $62\frac{1}{2}$
" $37\frac{3}{4}$
" $19\frac{4}{8}$
" $17\frac{7}{16}$

$137\frac{3}{4\frac{7}{0}}$

Add $\frac{1}{2}$, $\frac{1}{4}$, $\frac{4}{8}$, and $\frac{7}{16}$ together, (say dollars.)

By Reduction $\frac{1}{2}$ is equal to 50 cents.
$\frac{1}{4}$ " 25 "
$\frac{5}{8}$ " $62\frac{1}{2}$ "
$\frac{7}{16}$ " $43\frac{3}{4}$ "

Ans. $1\frac{1}{16} = \$1.81\frac{1}{4}$

Add	$\$\frac{1}{2}$	Add	$\$\frac{1}{2}$	Add	$\$\frac{3}{16}$
"	$\frac{1}{16}$	"	$\frac{7}{8}$	"	$\frac{1}{2}\frac{3}{8}$
"	$\frac{3}{4}$	"	$\frac{3}{16}$	"	$\frac{3}{16}$

CASE X.

To add mixed or compound fractions.

1. Add $\frac{3}{4}$ of a day $\frac{1}{4}$ of an hour, and $\frac{3}{16}$ of a minute together? Ans. 16h. 48m. 18s.

2. Add $\frac{1}{3}$ of a year, $\frac{1}{4}$ of a month, $\frac{1}{7}$ of a week, $\frac{1}{3}$ of a day, $\frac{1}{12}$ of an hour, and $\frac{1}{4}$ of a minute together?
Ans. 4m. 1w. 1d. 8h. 5m. 48s.

3. Add $\frac{1}{2}$ of an eagle, $\frac{3}{4}$ of a dollar, $\frac{7}{16}$ of a dime, and $\frac{1}{2}$ of a cent? Ans. $\$8.82\frac{1}{2}$.

4. Add $\frac{1}{3}$ of a week, $\frac{1}{4}$ of a day, and $\frac{1}{3}$ an hour together? Ans. 2d. $14\frac{1}{2}$h.

5. Add $\frac{3}{4}$ of a dollar, $\frac{2}{5}$ of a dollar, and $\frac{1}{2}$ of a dime together? Ans. $\$1.45\frac{1}{2}$.

6. Add $\frac{1}{3}$ of a yard, $\frac{1}{3}$ of a foot, and $\frac{1}{3}$ of an inch together? Ans. 1 ft. 4 in. 1 barley corn.

CASE XI.

To add compound fractions together, connected by the preposition OF (see Def. 9.)

GENERAL RULE.

Multiply the numerators together for a new numerator, and the denominators together for a new denominator. Reduce the fractions, and then add them together agreeably to Case VIII. or IX.

1. *Example.*—Add $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$, and $\frac{1}{3}$ of $\frac{2}{3}$ of $\frac{1}{4}$ together? Ans. $\frac{1}{4}\frac{7}{2}$.

Operation, $\frac{4}{5} \times \frac{5}{6} \times \frac{3}{4} = \frac{60}{120}$ reduced is $\frac{1}{2}$. Now, it is plain, that $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{3}{4}$ of the first compound is equal to $\frac{1}{2}$, and $\frac{7}{9} \times \frac{2}{3} \times \frac{1}{4}$ of the second compound is equal to $\frac{14}{108}$ reduced is equal to $\frac{7}{54}$, which added to $\frac{1}{2}$ the sum is $\frac{17}{27}$ as required.

2. How much is $\frac{1}{4}$ of $\frac{1}{5}$ of a dollar? Ans. 5c.

3. How much is $\frac{3}{5}$ of $\frac{3}{10}$ of a dollar? Ans. $\frac{9}{50}$ or 18c.

4. How much is the $\frac{1}{2}$ of $\frac{2}{3}$, the $\frac{1}{2}$ of $\frac{3}{4}$, and the $\frac{1}{2}$ of $\frac{3}{5}$ of a dollar? Ans. $1\frac{1}{120}$ or \$1.00c. $8\frac{1}{3}$m.

5. Add $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{4}{5}$ of a dollar, to $\frac{1}{5}$ of $\frac{7}{8}$ of $\frac{4}{5}$ of a dollar? Ans. \$$\frac{6}{25}$ or 24c.

Operation, $\frac{1}{2} \times \frac{1}{4} \times \frac{4}{5} = \frac{4}{40} = \frac{1}{10}$ of a dollar or 10c.

And $\frac{1}{5} \times \frac{7}{8} \times \frac{4}{5} = \frac{28}{200} = \frac{7}{50}$ of a dollar or 14c.

Adding fractions together, \$$\frac{12}{50}$ = 24c.

1. How much is the $\frac{1}{3}$ and $\frac{1}{2}$ of $\frac{1}{8}$ of a dollar? Ans. 50c.
2. How much is the $\frac{1}{4}$ and $\frac{1}{2}$ of $\frac{4}{5}$ of a dollar? Ans. 65c.
3. How much is $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ of a yard? Ans. 1 ft. 3 in.
4. How much is $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{1}{5}$ of \$5.00? Ans. $12\frac{1}{2}$c.
5. How much is the $\frac{1}{2}$ and $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{1}{6}$ of a year? " 7m.
6. How much is the $\frac{1}{20}$ and $\frac{4}{5}$ of $\frac{1}{10}$ of $\frac{7}{8}$ of $\frac{3}{4}$ of an Eagle? Ans. \$$1.02\frac{1}{2}$.

CASE XII.

To reduce mixed fractions to parts, or to an improper fraction. (See 11th Definition.)

RULE.—Multiply the whole number by the denominator of the fraction, and add the numerator to the product for the numerator of the fraction sought, under which will be the given denominator.

Example.—Reduce $17\frac{1}{2}$ dollars to half dollars.

ILLUSTRATION.

It is well known that two half dollars are equal to one dollar; consequently, as 1 dollar is = to 2 halves, 17 units or 17 dollars will contain 17 times as much, to which if we add one-half we get 35 halves for the required answer.

1. Bring $19\frac{3}{4}$ to quarters? Ans. $\frac{79}{4}$
2. Bring $20\frac{1}{4}$ to quarters? " $\frac{81}{4}$
3. Bring $33\frac{1}{3}$ cts. to thirds? " $\frac{100}{3}$
4. Bring $16\frac{7}{8}$ to eighths? " $\frac{135}{8}$
5. Bring $87\frac{1}{2}$ to halves? " $\frac{175}{2}$
6. Bring $14\frac{6}{7}$ to an improper fraction? " $\frac{104}{7}$

TO MULTIPLY FRACTIONS.

CASE I.

When the fractions are proper.

RULE.—Multiply the numerators together for a new numerator, and the denominators together for a new denominator.

ILLUSTRATION.

It is manifest, that when a number is multiplied by 1, the product is equal to the multiplicand; therefore, when a number is multiplied by a fraction, which is less than 1, the product must be less than the multiplicand.

Example 1.—Multiply $\frac{1}{2}$ by $\frac{1}{2}$? Ans. $\frac{1}{4}$.

From the analysis of Geometry, we find, that if a line be divided into 2 equal parts, the square of the whole line is 4 times the square of half the line: thus, let the line A $\frac{1}{2}$ | $\frac{1}{2}$ B be one mile, yard, &c. The square of 1 is 1, because 1×1 is 1, and $\frac{1}{2}$ squared is $\frac{1}{4}$, hence, $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ of 1.

CASE II.

When the multiplier and multiplicand are both mixed numbers.

RULE.—Bring them to improper fractions, agreeably to Case XII. (Addition,) this done, multiply the numerators together as before, for a new numerator, and the denominators together for a new denominator; divide the new numerator (so called) by the new denominator, and the result will be the product of the mixed numbers.

ILLUSTRATION.

In the rectangular or parallelogram A B C D, the length of the side A B is 10½ yards, and the length of the line A C is 7½ yards, the line A B is divided into 21 parts, and the line A C into 15 equal parts, which are drawn at right angles to each other, consequently, there are 315 rectangles in the whole figure A B C D, and every four of these make 1 square yard, this is manifest from the following example: therefore, $10\frac{1}{2} \times 7\frac{1}{2} = \frac{21}{2} \times \frac{15}{2} = \frac{315}{4} = 78\frac{3}{4}$ as required.

CASE III.

To multiply a whole number by a fraction.

RULE.—Multiply the whole number by the numerator of the fraction, and divide the product by the denominator, the quotient will be the result.

From what has been already stated, it is evident, that the multiplication of a whole number by a fraction implies the taking some part of it; for instance, if we multiply 4 by ½, agreeably to the rule $4 \times \frac{1}{2} = \frac{4}{2} = 2$, and $9 \times \frac{1}{3} = 3$, &c.

Multiply	35 by ½.	Multiply	84 by ¼.
"	39 by ⅓.	"	96 by ¼.
"	72 by ⅓.	"	80 by ½.

APPLICATION OF FRACTIONS TO 'SHORT ACCOUNTS.'

1. Multiply 11½ by 11½ cts. Example, $\frac{23}{2} \times \frac{23}{2} = \frac{529}{4}$ Ans. $1.32¼.
2. What will 7⅛ lbs. come to at 8½c. pr. lb? Ans. 61⅝c.
3. What will 4½ lbs. come to at $\frac{1}{8}$ per lb? Ans. 56¼c.
4. What will 19¾ yards come to at $\frac{3}{8}$ of a dollar? Ans. $7.40⅝.
5. What will 2¾ yards come to at $\frac{7}{8}$ of a dollar? Ans. $2.40⅝.
6. What will 6⅞ lbs. of tea cost at 65¾ cts. per pound? Ans. 4.52\frac{1}{32}$.

SUBTRACTION OF FRACTIONS.

CASE I.

If the fractions have a common denominator.

RULE.—Subtract the lesser numerator from the greater, and under the remainder write the common denominator, and reduce the fraction if necessary.

1. Example—From $\$\frac{3}{4}$ take $\frac{1}{4}$? Ans. $\frac{1}{2}$ of a doll. or 50c.
$3 - 1 = \frac{2}{4} = \frac{1}{2}$, or $.75 - .25 = .50$.
2. From $\$\frac{5}{8}$ take $\frac{3}{8}$? Ans. $\$\frac{2}{8}$ or .25c.
3. From $\$\frac{7}{10}$ take $\frac{2}{10}$? 5. From $\$\frac{9}{10}$ 6. From $\frac{7}{8}$
4. From $\$\frac{1}{2}$ take $\frac{1}{4}$? Take $\$\frac{5}{10}$ Take $\frac{5}{8}$

CASE II.

If the denominators of the fractions are unlike.

RULE.—Find a common denominator according to Case VI. Addition, ("Second Method.")

1. Example—From $\frac{10}{11}$ take $\frac{4}{7}$? Ans. $\frac{26}{77}$.

Here the denominators of the fractions are in the ratio of 11 to 7, then $\frac{10}{11} \times 7 = \frac{70}{77}$
and $\frac{4}{7} \times 11 = \frac{44}{77}$ $70 - 44 = \frac{26}{77}$.

By Case I, or by Case VI. Addition, find a common denominator; thus, by cross multiplication.

$\frac{10}{11} \times \frac{4}{7}$ $10 \times 7 = 70$
$11 \times 4 = 44$

$11 \times 7 = 77$ common denominator, the result is $\frac{26}{77}$.

1. From $\$\frac{9}{10}$ take $\frac{7}{8}$? Ans. $\$\frac{1}{80}$ or .02½c.
2. From $\$\frac{5}{8}$ take $\frac{1}{2}$? " $\$\frac{1}{2}$ or .50c.
3. From $\$1\frac{5}{6}$ take $\frac{3}{4}$? (Here the ratio is as 4 to 1.)
4. From $\$1\frac{5}{6}$ take $\frac{8}{10}$? Ans. $\frac{3}{4}$ or .75c.
5. From $\$\frac{3}{4}$ take $\frac{2}{5}$? " $\frac{3}{20}$ or .15c.
6. From $\$\frac{3}{4}$ take $\frac{3}{10}$? " $\frac{9}{20}$ or .45c.

CASE III.

When the fractions have a unit for a numerator.

RULE.—Write the difference of their denominators over their product.

1. Example—From $\frac{1}{4}$ take $\frac{1}{4}$? Operation, $\frac{4-3}{4\times3}=\frac{1}{12}$.
2. From $\frac{1}{7}$ take $\frac{1}{7}$? Ans. $\frac{4}{7}$.
3. From $\frac{1}{5}$ take $\frac{1}{6}$? Ans. $\frac{1}{6}$. From $\frac{1}{4}$ take $\frac{1}{5}$? " $\frac{1}{20}$.
4. From $\frac{1}{2}$ take $\frac{1}{3}$? Ans. $\frac{1}{6}$. From $\frac{1}{5}$ take $\frac{1}{8}$? " $\frac{3}{40}$.

CASE IV.

When the numerators are alike and more than a unit.

RULE.—Multiply the difference of the denominators by one of the numerators for a new numerator. Then multiply the denominators together for a new denominator.

Note.—This Rule is general, except in cases of compound fractions. (☞ See Case V.)

Example.—From $\frac{3}{4}$ take $\frac{3}{5}$.

Operation, $5-4=1\times3=$ 3 new numerator.
$5\times4=$ 20 new denominator.

Or thus: $\frac{3}{4}\times5=\frac{15}{20}$
$\frac{3}{5}\times4=\frac{12}{20}$ difference $\frac{3}{20}$ answer.

CASE V.

From a compound fraction to take a mixed fraction.

Example 1.—From $\frac{1}{2}$ of 12, take $7\frac{3}{4}$? Ans. $1\frac{7}{20}$.

Operation. (According to Case XI. Addition, in relation to mixed and compound fractions connected by the preposition OF.) Multiply the numerators together for a new numerator, and the denominators together for a new denominator.

Thus: $\frac{1}{2}\times\frac{12}{1}=\frac{12}{2}=9\frac{3}{5}$
Then from $9\frac{3}{5}$ take $7\frac{3}{4}$
Operation, $\frac{3}{5}\times4=\frac{12}{20}$.
$\frac{3}{4}\times5=\frac{15}{20}$.

We cannot take 15 from 12, but 15 from the common denominator 20, and 5 remains, 5 and 12 are 17; set down $\frac{17}{20}$ and carry 1 to 7, which make 8, then 8 from 9 and 1 remains, which set down before the fraction, thus: $1\frac{17}{20}$.

1. From $\frac{3}{4}$ of $8\frac{1}{2}$ take $\frac{3}{5}$ of 5? Ans. $1\frac{1}{4}\frac{1}{0}$.
2. From $\frac{1}{4}$ of 3 take $\frac{1}{2}$ of 2? " $\frac{1}{1}\frac{1}{2}$.

CASE VI.

To subtract a proper fraction from a whole number.

RULE.—Subtract the numerator of the fraction from the denominator, and under the remainder place the denominator, and carry 1, to be subtracted from the minuend.

Example—From \$10 take $\frac{3}{8}$ of a dollar.

10
$0\frac{3}{8}$

\$9$\frac{5}{8}$ It is plain, that if we take $\frac{3}{8}$ of a dollar, from a whole that $\frac{3}{8}$ will remain.

Thus, from \$10.00
take 37$\frac{1}{2}$

which is equivalent to $\frac{3}{8}$, \$9.62$\frac{1}{2}$ or $\frac{5}{8}$ answer.

DIVISION.

To generalize division of fractions, the *dividend* must be considered as having the same relation to the *quotient* that the *divisor* has to *unity*, because the divisor and quotient are the two factors of the dividend; when for instance, the divisor is 5, the dividend is equal to 5 times the quotient, and consequently, this last is the fifth part of the dividend. If the divisor be a fraction, suppose $\frac{1}{2}$, the dividend cannot be but half the quotient, or the latter must be double of the former.

The definition just given easily suggests the mode of proceeding when the divisor is a fraction. Let us take for example $\frac{1}{5}$, in this case the dividend ought to be only $\frac{1}{5}$ of the quotient, but $\frac{1}{4}$ being $\frac{1}{4}$ of $\frac{1}{5}$ we shall have $\frac{1}{4}$ of the quotient, because $\frac{1}{4} \times \frac{1}{5} = \frac{1}{2}\frac{1}{0} = \frac{1}{5}$ reduced by taking $\frac{1}{4}$ of the dividend, or dividing by 4. By having $\frac{1}{4}$ of the quotient, we have only to multiply it by 5, to attain it:

thus, $\frac{1}{4} \times 5 = \frac{5}{4} = 1$ the quotient. In this operation, the dividend is divided by 4, and multiplied by 5, which is exactly the same as taking $\frac{1}{4}$ of the dividend or multiplying by $\frac{5}{1}$, which fraction is no other than the divisor inverted. Q. E. D.

From whence, the following general rule is derived.

CASE I.

To divide a whole number, or a fraction by a fraction.

RULE.—Multiply the whole number, or fraction by the divisor inverted.

Example. Divide 9 by $\frac{3}{4}$.

Operation. 9 is equal to $\frac{9}{1} \times \frac{4}{3} = \frac{36}{3} = 12$.

CASE II.

If there be whole numbers joined to the given fractions.

RULE.—Reduce them to improper fractions, and invert the divisor according to the general rule.

Example.—Divide 9 by $\frac{3}{4}$?	Ans. $\frac{45}{2}$ or $22\frac{1}{2}$.
" 18 by $\frac{6}{7}$?	Ans. 15.
" 18 by $\frac{6}{8}$?	" $21\frac{3}{5}$.
" $7\frac{1}{2}$ by $\frac{1}{3}$?	" $22\frac{1}{2}$.
" $\frac{98}{9}$ by $\frac{28}{63}$?	" 49.
" $\frac{11}{11}$ by $\frac{11}{11}$?	" 1.
" \$14 by \$$\frac{2}{3}$?	" 35.
" \$22 by \$$\frac{2}{3}$?	" 55.

RATIO OF MIXED NUMBERS.

The following questions for exercise are well calculated to exercise the learner in addition and multiplication of fractions.

1. Find 2 numbers in a given ratio, as 5 is to 6, so that their sum and product may be equal?

EXAMPLE.

Operation, $5 + 6 = \frac{11}{5} = 2\frac{1}{5}$ and $\frac{11}{6} = 1\frac{5}{6}$; consequently, $2\frac{1}{5}$ and $1\frac{5}{6}$ are in the ratio of 5 to 6.

Operation, $2\frac{1}{5}$ added to $1\frac{5}{6} = 4\frac{1}{30}$

and $2\frac{1}{5}$ multiplied by $1\frac{5}{6} = 4\frac{1}{30}$ as required; hence, the ratio of numbers can be proved as above, as well as mixed numbers, whose sum and product may be equal.

2. Find 2 numbers in a given ratio, as 3 to 5, whose sum and product are equal? Ans. $2\frac{2}{3}$ and $1\frac{9}{5}$.

3. Find 2 numbers in the ratio of 5 to 9, whose sum and product are equal? Ans. $1\frac{5}{9}$ and $2\frac{4}{5}$.

4. Find 2 numbers in a given ratio as 2 to 3, whose sum and product may be equal? Ans. $1\frac{2}{3}$ and $2\frac{1}{2}$.

5. Find 2 numbers in the ratio of 3 to 8, whose sum and product are equal? Ans $1\frac{3}{8}$ and $3\frac{16}{24}$.

RULE OF THREE DIRECT IN FRACTIONS.

1. If $\frac{1}{4}$ of a yard of velvet cost $62\frac{1}{2}$ cts. what will $2\frac{7}{8}$ yards cost?

Operation, $\frac{4}{1} \times \frac{125}{2} \times 2\frac{3}{8} = \frac{11500}{16}$

The Rule for the common method is to invert the first term, then after preparing the fractions, to multiply the numerators together for a new numerator, and the denominators for a new denominator.

OR THUS:

Let a line be drawn in all statements representing *equality*, placing *multipliers* on the right, and *divisors* on the left, and transpose the numerators when cancelled on both sides. This is a general principle in all proportional operations.

$$4)\ \frac{1}{4}\ 1 \quad \frac{125}{2} \qquad \frac{125 \times 23}{4} = \frac{2875}{4} = 7.18\tfrac{3}{4}.$$

$$2 \quad \frac{23}{8}\ 2 \qquad \text{Answer as above.}$$

2. At $18\frac{3}{4}$ cts. per pound, what will $63\frac{1}{2}$ lbs. come to? Ans. $\$11.90\frac{3}{8}$.

3. If $\frac{1}{3}$ of a lb. of cinnamon bring $\frac{4}{7}$ of a dollar, what will $1\frac{3}{5}$ lbs. cost? Ans. $\$2.74\frac{2}{7}$.

INVERSE PROPORTION BY FRACTIONS.

RULE.

When the fractions are prepared and the third term inverted for a divisor, as in division of fractions, then agreeably to the Rule in multiplication of fractions, multiply the numerators for a new numerator, and the denominator for a new denominator.

1. How many yards of brown Holland, 5 quarters wide will line 20 yards, that is 3 quarters wide? Ans. 12 yds.

2. How many yards of matting 2 feet, 6 inches broad, will cover a room that is 27 feet long and 20 feet broad? Ans. 72 yds.

3. How much shalloon $\frac{3}{4}$ of a yard wide, will line $4\frac{1}{2}$ yards of cloth $1\frac{1}{2}$ yards wide? Ans. 9 yds.

4. What quantity of shalloon $\frac{3}{4}$ yard wide, will line $7\frac{1}{2}$ yards of cloth $1\frac{1}{4}$ yards wide? Ans. 15 yds.

5. If 3 men can do a piece of work in $4\frac{1}{2}$ hours, in how many hours will 10 men do the same? Ans. $1\frac{7}{10}$.

6. How many pieces of cloth at $20\frac{1}{8}$ dollars per piece, are equal in value to $240\frac{1}{4}$ pieces at $12\frac{1}{2}$ dollars per piece? Ans. $149\frac{11}{17}\frac{17}{17}$ pieces.

PROMISCUOUS EXAMPLES.

1. What will $\frac{1}{4}$ of $2\frac{1}{2}$ cwt. of chocolate come to when $6\frac{1}{4}$ lbs. cost $\frac{2}{3}$ of a dollar? Ans. $\$10.76\frac{1}{3}$.

2. If $\frac{7}{8}$ of a yard of cloth cost $\$2\frac{5}{8}$, what will $5\frac{5}{7}$ yards cost at the same rate? Ans. $\$16.25$.

3. If $4\frac{1}{2}$ of a yard cost $\$9.75$, what will $13\frac{1}{2}$ yards cost? Ans. $\$29.25$.

4. If $\$1.75$ will buy 7 lbs. of loaf sugar, how much will $\$213.50$ buy? Ans. 7 cwt. 2 qrs. 14 lbs.

5. How many yards of carpeting that is half a yard wide, will cover a room that is 30 feet long and 18 feet wide? Ans. 120 yds.

LECTURE IX.

DIRECT PROPORTION.

(Sign : : : :)

PROPORTION shews the direct relation of one object or thing to another, as to comparison or symmetry, viz: *form*, *size*, *length*, *breadth*, *depth*, *rate*, *price*, &c. But in relation to Arithmetic or Geometry, it has a determinate meaning. EUCLID has proven his Theory of proportions in the fifth book of his Elements by demonstrating its principles, relations, and application to lines; therefore, it is apparent, that we thence derive the name of Geometrical Proportion. In Direct Proportion, the first term is to the second as the third term is to the fourth, that is, as 2 is to 4, so is 8 to 16, written thus: 2 : 4 : : 8 : 16, these numbers are proportional, because, agreeably to the 16th proposition of Euclid, Lib. 6, the rectangle or product of the extremes is equal to the product of the means; hence, $2 \times 16 = 4 \times 8 = 32$ or $\frac{1}{2} = \frac{8}{16} = 2$. From this it is plain, that Proportion is the combination of two equal ratios, and that there are two antecedents and two consequents.

Note to Teachers.—Require the learner to recite the answers to the following questions.

Q. Why is this called the rule of Proportion?

A. Because it shews the combination of two equal ratios.

Q. What is the first term of a proportion called?

A. The antecedent.

Q. What is the second term called?

A. The consequent.

Q. If 2 yards of muslin cost 25 cents, what will 6 yards cost? Ans. 75c.

Q. Why is the second term (25 cts.) called the first consequent?

A. Because it is the value or cost of the antecedent.

Q. How are the terms arranged?

A. As the first term (or antecedent) is to the second (its consequent) so is the third, (or antecedent) to the fourth, (or its consequent.)

Q. How is the operation performed?

A. Multiply the second and third terms together, and divide the product by the first.

Q. What is the first antecedent called?

A. An antecedent of the first relation.

Q. What is the second antecedent called?

A. An antecedent of the second relation.

Q. When is a question stated correctly?

A. When the first and third terms can be brought to the same denomination.

Q. What is the general rule for stating?

A. When a question is written thus: for instance, at 10 cts. per lb., what will 10 lbs. cost? or if 1 lb. cost 10 cts. what will 10 lbs. cost? In either case, the first term is 1 lb., then say as the first antecedent is to its consequent, so is the second antecedent to its consequent.

Q. When the commencement of a question is written with the words, *what is the value; how much*, *bought*, or *sold;* what is the rule?

A. In either case, the first object or thing mentioned in the question will be the third term.

Q. What terms of a proportion can be contracted or reduced?

A. The first and third, or first and second.

Q. Why except the second and third terms?

A. Because the means do not shew a proper relation to each other as to cost or value; hence, it is evident, that the second or middle term must be of the same name or denomination with the answer (or 'demand.')

Q. What is meant by relation as applied to proportion?

A. Nothing more than the quotient of a division.

1. If 3 lbs. of sugar cost 24 cents, what will 10 lbs. cost? Ans. 80c.

lbs. cts. lbs.

As 3 : 24 : : 10 = 80c. or by contraction,

As 1 : 8 : : 10 = 80c.

1 lb. and 10 lbs. are the antecedents.

8 cts. and 80 cts. the consequents.

The relative proportion of the antecedents and consequents is the same. Thus, $\frac{1}{3}$ cts. = $\frac{10}{30}$ cts. = 8.

2. If 3 yards of cloth cost $9, what will 12 yards cost? Ans. $36.

3. What is the value of 9.7 lbs. of silver at $1.5 per ounce? Ans. $174.60.

4. What will 240 bushels of wheat amount to at the rate of $6 for 5 bushels? Ans. $288.

5. How much will 17 cwt. 3 qrs. 14 lbs. of iron cost at $4.75 per cwt.? Ans. 84.90\frac{5}{8}$.

6. Sold 120 bushels of corn for $54, how much did it cost per bushel? Ans. 45c.

7. Bought 29 yds. of muslin for 10.87\frac{1}{2}$, how much is it per yard? Ans. 37$\frac{1}{2}$c.

8. Bought 2 loads of corn, one containing 75 bushels and the other 87 bushels, at 52 cents per bushel, what is the amount? Ans. $84.24.

9. Bought 3 pipes of brandy containing 120$\frac{1}{4}$, 124$\frac{3}{4}$, and 123$\frac{1}{4}$ gallons at 43$\frac{3}{4}$ cts. per gallon, how much is the amount? Ans. 161.21\frac{7}{8}$.

10. If a staff 4 feet long cast a shade on level ground 7 feet long, what is the height of a steeple whose shade at the same time is 198 feet? Ans. 113$\frac{1}{7}$.

11. An express who rides from Washington city had been dispatched at the rate of 60 miles per day for five days, when a second was sent to overtake him, in order to do which, he must travel 75 miles a day, in what time will he overtake the former? Ans. in 20 days.

12. A prize of $2,329 was divided between two persons, A. and B. whose shares therein were in proportion as 5 to 12, what was the share of each?

Ans. A. $685, and B. $1644.

13. When a man's yearly income is $949, how much is it per day? Ans. $2.60.

14. Bought a stove weighing 4 cwt. 3 qrs. 24 lbs. at $2.10 per cwt. and 27 lbs. of pipe at 18$\frac{3}{4}$ cts. per lb. with 2 elbows at 50 cts. each, what is the price of the stove pipe and elbows? Ans. 16.48\frac{3}{4}$.

15. Bought 4 pieces of linen, viz: No. 1 and 2, each contained 27$\frac{1}{4}$ yards, No. 3 and 4, contained each 25$\frac{3}{4}$ yards at 62$\frac{1}{2}$ cts. per yard. what was the cost?

Ans. 66.56\frac{1}{4}$.

16. If a person's salary be $1,333 per annum, and his daily expences $2.14, how much will he save?

Ans. $551.90.

17. What will 4 pieces of sattinet, containing 23, 24, 25, and 27 yards come to at 72 cents per yard?

Ans. $71.28.

18. A farmer upon measuring his corn produced by a certain field, found he had but 48 bushels. It appeared that it yielded one third more than was sown, how much was that? Ans. 36.

19. A bookseller sold 10 books at a certain price, and afterwards 15 more at the same rate; now at the latter sale he received $2.25 more than at the former, what did he receive for each book? Ans. 45c.

CONTRACTED OPERATIONS.

CASE I.

If the first term be a multiple or part of the second, the third will be a multiple or part of the fourth.

Example 1.—If 6 yards cost $18, how much will 24 yards cost? Ans. $72.

CASE II.

If a part of the first be added to or subtracted from the first so as to be equal to the second, a like multiple must be added to or subtracted from the third.

Example 1.—If 12 yards of cloth cost $15, what will 20 yards cost? Ans. $25.

2. If 6 yards cost $9, what will 24 yards cost?

Ans. $36.

Solution.—Agreeably to Case II. the difference between the first and second terms is 3, and as 3, is the half of 6, ½ of the third term must be added to it. Thus, $24 + {}^{24}/_{2} = \$36$ as above.

3. If 8 yards cost $12, what will 48 yards cost?

Ans. $72.

4. If 45 yards cost $30, what will 165 yards cost?

Ans. $110.

5. If 3 yards cost $2, what will 27 yards cost?

Ans. $18.

6. If 9 yards cost $6, what will 30 yards come to?

Ans. $20.

7. If 3 yds cost $6, what will 10 yds. cost? " $20.

8. Bought 6 yards for $8, how much did 30 yards cost? Ans. $40.

INVERSE PROPORTION.

It is generally admitted by writers on Arithmetic that more requires less, or less requires more; more requires less, when the third term is greater than the first, and the fourth less than the second. 2d. Less requires more when the third term is less than the first, and the fourth

term greater than the second. These definitions may be illustrated by the following questions, viz:

1. If in 10 days 8 men can perform a piece of work, in what time could 40 men do the same.

men. days. men.

Solution.—As 8 : 10 : : 40 $= \frac{1 \times 10}{5} = 2.$

Illustration—8 men at work 10 days uniformly would perform as much as 80 men in 1 day. Consequently, if 80 men perform the same in 1 day, 40 men will do it in 2 days. Q. E. D.

2. If 8 men can perform a piece of work in 10 days, how many men would be required to perform the same in 2 days? Ans. 40.

days. men. days.

Solution—As 10 : 8 : : 2 : 40.

Notwithstanding, these statements are made to correspond with the above definitions, and bring correct answers. Yet, it is manifest, they are wrong!!! if the doctrine of proportion be recognized, for agreeably, to Prop. B. book 5 Euclid, in Inverted Proportion, the third is to the second, as the first is to the fourth. In solution 1st, we find that more requires less for 10 days require 2 days, and in solution 2d, less requires more—for 8 men require 40 men.

Now the proper method of stating is thus: as 40 men, : 10 days : : 8 men = 2 days.

Solution 2d—2 days : 8 men : : 10 days = 40 men.

Hence, $40 \times 2 = 10 \times 8$,

And $2 \times 40 = 8 \times 10$—for by Prop. 16, 6 Euclid, the rectangle of the extremes will be equal to that of the means, which is not the case with the first and second statements; therefore, it follows, that although custom and the opinion of most writers, sanction the truth of the 'first and second' statements they are evidently mistaken.

3. Suppose 450 soldiers are in a garrison, and their provisions are calculated to last them but 5 months, how many must leave the garrison, that the same allowance may be sufficient for those who remain 9 months.

Ans. 200.

4. If a man perform a piece of work in 15 days of 12 hours long, in how many days of 10 hours long can he perform the same? Ans. 18 days.

5. There is a cistern having a pipe that will empty it in 6 hours, how many pipes of the same capacity will empty it in 20 minutes? Ans. 18 pipes.

COMPOUND PROPORTION.

Is that, in which 5 terms are given to find the *sixth*. Three of those terms are a supposition and two a demand.

RULE. Arrange the terms so that two terms, one side of the statement shall be of the same name and denomination with two terms on the other side. Consequently, the demand will be in the middle term.

Example.—If 10 men in 18 days earn $56, how many dollars can 20 men earn in 36 days?

	men.	*demand.*	*men.*	
Mode of operation	10	$56	10 20 2	by contraction
days	18	4	18 36 2	
			4	

1)224

$224.00 Answer.

2. If 7 reapers get $21 for 3 days work, how many reapers will earn $96 in 32 days? Ans. 3 reapers.

3. If a family of 8 persons in 24 months spend $480, how much would 16 persons spend in 8 months.

Ans. $320 00.

4. If 4 men mow 96 acres in 12 days, how many acres can 8 men mow in 16 days? Ans. 256 acres.

5. If 4 men receive $24 for 6 days work, how much will 8 men receive for 12 days work. Ans. $96.

PRACTICAL ARITHMETIC.

AS ABBREVIATED IN THE COUNTING ROOM.

OF CURRENCIES AND COINS.

Parts of a dollar.	*cents.*	*Of a dollar.*	*s. d.*	*Parts of a Pound Sterling.*	*s. d.*	*Parts of a Pound.*	6*d.*	*Parts of a Shilling.*
	50 $\frac{1}{2}$		10.0 $\frac{1}{2}$		1.8 $\frac{1}{12}$		4 $\frac{1}{2}$	
	25 $\frac{1}{4}$		5.0 $\frac{1}{4}$		1.0 $\frac{1}{20}$		3 $\frac{1}{4}$	
	20 $\frac{1}{5}$		6.8 $\frac{1}{3}$		0.6 $\frac{1}{30}$		2 $\frac{1}{6}$	
	12$\frac{1}{2}$ $\frac{1}{8}$		4.0 $\frac{1}{5}$		0.4 $\frac{1}{60}$		1 $\frac{1}{12}$	
	6$\frac{1}{4}$ $\frac{1}{16}$		3.4 $\frac{1}{6}$		0.3 $\frac{1}{80}$			
	10 $\frac{1}{10}$		2.6 $\frac{1}{8}$					

OF WEIGHTS.

Weights. *Parts of a Ton.*	Weights. *Parts of a cwt.*	Weights. *Parts of a cwt.*	*Weights.*
20 cwt. 1 ton	4 qrs. 1 cwt	To take	16lb. $\frac{1}{7}$
10 $\frac{1}{2}$ "	2 " $\frac{1}{2}$ "	parts of 3	14 " $\frac{1}{8}$
5 $\frac{1}{4}$ "	1 " $\frac{1}{4}$ "	qrs of a cwt.	8 " $\frac{1}{14}$
4 $\frac{1}{5}$ "	28 lbs. 1 qr.	2 qrs. $\frac{1}{2}$	7 " $\frac{1}{16}$
2 $\frac{1}{10}$ "	14 " $\frac{1}{2}$ "	1 " $\frac{1}{2}$	
1 $\frac{1}{20}$ "	7 " $\frac{1}{4}$ "	of 2 qrs.	

OF MEASURES.

Measures. Parts of a yd	*Measures.* Parts of a bu.	*Measures.* Parts of a gal.	*Measures.* Parts of an ac	*Measures.* Parts of an acre.
4 qr 1 yd	4 p. 1 bu.	4 qts. 1 gl	160 ps 1 a.	4 rds. 1 a.
2 " $\frac{1}{2}$ "	2 " $\frac{1}{2}$ "	2 " $\frac{1}{2}$ "	80 " $\frac{1}{2}$ "	2 " $\frac{1}{2}$ "
1 " $\frac{1}{4}$ "	1 " $\frac{1}{4}$ "	1 " $\frac{1}{4}$ "	40 " $\frac{1}{4}$ "	1 " $\frac{1}{4}$ "
nails	quarts. pecks	pints.	20 " $\frac{1}{8}$ "	perches,
4 $\frac{1}{4}$ of yd	8 1	2 1 qt.	10 " $\frac{1}{16}$ "	20 $\frac{1}{2}$ rood.
2 $\frac{1}{8}$ "	4 $\frac{1}{2}$	1 $\frac{1}{2}$ "	5 " $\frac{1}{32}$ "	10 $\frac{1}{4}$ "
1 $\frac{1}{16}$ "	2 $\frac{1}{4}$	$\frac{1}{2}$ $\frac{1}{4}$ "	32 " $\frac{1}{5}$ "	8 $\frac{1}{5}$ "
	1 $\frac{1}{8}$			
	pts. of gal.			
	2 = $\frac{1}{2}$			

CASE I.

How to take parts for $\frac{3}{4}$ of a cent, $\frac{3}{4}$ of a yard, $\frac{3}{4}$ of a ton, $\frac{3}{4}$ of a cwt. $\frac{3}{4}$ of a year, &c.

RULE.—Take the aliquot parts for $\frac{1}{2}$ first, and then for $\frac{1}{4}$, and add the product together; or, multiply the given quantity by 3, and divide by 4. If it be $\frac{1}{4}$, divide by 4, for $\frac{1}{4}$, divide by 2.

CASE II.

When the given price is dollars and cents.

RULE.—Multiply the given quantity by the dollars, and take aliquot parts of the cents, and add the products together.

CASE III.

When the given quantities are of various denominations, as cwts. qrs. and lbs.; yards, quarters, nails; or bushels, pecks, and quarts, &c.

RULE.—Set down the price per cwt., multiply it by the number of cwts. given, and take aliquot parts, for the quarters and lbs. &c.

Q. What is the amount of 15 cwt., 2 qrs, 7 lbs. of iron at $10 per cwt?

Operation $10.00
× 15

$150.00

2 qrs. = $\frac{1}{4}$ cwt. } 7.50 price of $\frac{1}{4}$ a cwt.
7 lbs. = $\frac{1}{8}$ of 2 qrs. } 0.93¾ price of 7 lbs.

$158.43¾

1. What will 36 yards of tape come to at $\frac{1}{4}$ of a ct. per yd.
2. " " 96 " " " $\frac{1}{4}$ "
3. " " 295 " " " $\frac{1}{2}$ "
4. " " 320 " " " $\frac{1}{2}$ "
5. " " 440 " " " $\frac{3}{4}$ "
6. " " 896 " " " $\frac{3}{4}$ "

CLOTH MEASURE.

		yds.	*qrs.*			$ *cts.*
1.	What is the value of	27	3	at $9.65	per yd.	267.79

		yds.	*qr.*	*nls.*		
2.	"	860	1	at 84 cts.		$722.61
3.	"	126	2	2 "	$4.75	601.47
4.	"	7	3	2 "	5.60	44.10
5.	"	29	1	3 "	3.75	110.39

DRY MEASURE.

		bush.	*pks.*			$ *cts.*
1.	What is the value of	120	2	at 35 cts.	per	42.17½

		bush.	*pks.*	*qts.*	*pts.*			
2.	"	780	3	2	0	at $1.17	per bush.	913.55
3.	"	1354	1	5	1 "	25	"	338.60½
4.	"	325	3	"	" "	62½	"	203.59½
5.	"	94	2	"	" "	87½	"	82.68¾

LIQUID MEASURE.

1. What will 784 gals come to at 84 cts per gal. $658.56

		gals.	*qts.*	*pts.*		*Ans.*	$ *cts.*
2.	"	765	3	1	at $2.18¾	per gal.	1675.35
3.	"	5 hhds.	31½	gals.	at $47.12	per hhd.	259.16
4.	"	17 "	15¾	or 3 qts.	63.75	"	1116.93¾
5.	"	428 gals.	3 qts.		at 1.40		600.25

LAND MEASURE.

		A.	*R.*	*P.*			$ *cts.*
1.	What is the value of	35	2	9	at $54.35		1935.54

		A.	*R.*	*P.*		
2.	"	146	3	10	at $35.10	5153 12
3.	"	750	1	4 "	12.25	9190.87
4.	"	175	3	12 "	52.15	9160.27¼
5.	"	196	2	32 "	44.10	8674.47
6.	"	189	3	39¾	49.99¾	9999.42

EXERCISES FOR THE SLATE.

Tons.	*cwt.*	*qr.*	*lbs.*			
19	19	3	27½	at $99.99¾	per ton,	Ans. $1999.93

Now 20 tons at $100 per ton is $2000.00

Deduct, 20 tons at ¼ cts. $0.05
Again, we find that
¼ lb. is $\frac{1}{4480}$ of a ton 4480) 99.9975(=0.02 0.07

As before $1999.93

What will 29⅞ yards come to at $4 per yd.

Operation 30 yds.—⅛=29⅞
30 yds. at $4=$120.00
Deduct ⅛ of $4.00 = 0.50
$119.50

		yds.					
2.	"	30⅝	at	$2.40	per	Ans.	$74.00
3.	"	19¾	"	1.75	"	"	34.56¼
4.	"	44⅜	"	0.80	"	"	35.50
5.	"	12⅝	"	3.15	"	"	39.77
6.	"	26¼	"	3.37½	"	"	88.59⅜

A merchant in Wilmington (Del.) bought the following bill of goods in Philadelphia for cash, and calculated to dispose of them, at 25 per cent profit.

			First cost.	
10 pieces Irish linen	236 yds.	at	44 cts.	$103.84
1 bale shirting muslin	400 "		15 "	60.00
19 pair shoes			1.75 "	33.25
12 " boots			2.25 "	27.00
50 yards white flannel			44 "	22.00
50 " ticking			20 "	10.00
120 " bombazine			45 "	54.00
				$310.09
To gain 25 per cent. add ¼				77.52¼
				$387.61¼

To MARK GOODS, assume some word containing 10 letters; suppose the word *Perthamboy*

Let these ten letters be represented by each of the nine

digits, with a cipher, viz: in order to obtain private marks for the retail price of the goods.—Thus,

The Irish linen was purchased for 44 cts.
To gain 25 per cent. add ¼ 11

55 retail price.

Hence the private mark is *hh*.

In like manner, the shirting cost 15 cts.
To gain 25 per cent., add ¼ 3¾

18¾ retail price.

Consequently it can be marked *pb r-t*.

Again, the shoes cost $1.75 per pair.
To gain 25 per cent. add 0.43¾

$2.18¾ selling price.

e-pb r-t, private mark.

The boots cost $2.25 per pair.
To gain 25 per cent. add 0.56¼

$2.81¼ selling price.

e.bp p-t, private mark.

The flannel cost 44 per yard.
To gain 25 per cent. add ¼ 11

55 selling price.

hh private mark.

The ticking cost 20 cts. per yard.
To gain 25 per cent. add ¼ 5

25 selling price.

eh private mark.

The bombazine cost 45 cts. per yard.
Add ¼ to gain 25 per cent. 11¼

56¼ selling price.

ha.p-t. private mark.

PROOF.

	yds.		marked.	
10 pieces Irish linen	236	at 55c.	(hh)	$129.80
1 bale shirting muslin	400	at 18¾	(pb r-t)	75.00
19 pair shoes at		$2.18¾	(e.pb r-t)	41.56¼
12 " boots at		2.81¼	(e.bp p-t)	33.75
50 yards ticking		0.25	(eh)	12.50
50 " flannel		0.55	(hh)	27.50
120 " bombazine		0.56¼	(ha p-t)	67.50
			As before,	$387.61¼

1. Bought 260 yards double milled cassimere at $3.00 per yd. Required the *private mark* at a gain of 20 per cent. profit. Ans. $936.00

Selling price $3.60, private mark "*r.ay.*"

2. Bought 50 pieces domestic muslin, each 29 yards, at auction, at 10 cents per yard, which I disposed of immediately at 10 per cent profit, what is the "*private mark,*" and the selling price? Ans. 11c.

And the private mark *pp.*

CASE III.

1. What is the price of 5 cwt. 1 qr. 14 lbs. at $2.50 per cwt.? Ans. $13.43¾.

	cwt.	qrs.	lbs.				
2.	10	2	7	at $10.25	per	cwt.	Ans. $108.26¼.
3.	7	3	19	at 4.15	"	"	32.86¼.
4.	16	2	0	at 5.18	"	"	85.47.
5.	0	2	14	at 27.10	"	"	16.93¾.

1. At 69 cents per yard, what will 60 yards three-quarters come to? Ans. $41.91¾.

2. At $1.46 per yard, what will 33 yards three-quarters come to? Ans. $49.27½.

3. At 43¾ cents per yard, what will 81 yards three-quarters come to? Ans. $35.77.

4. At $1.64 per yard, what will 104 yards, three-quarters, and 3 nails come to? Ans. $172.09¾.

5. At 33⅓ cents per yard, what will 63 yards of muslin come to? Ans. $21.00.

6. At 31¼ cents per yard, what will 40 yards come to? Ans. $12.50.

7. At 93¾ cents per yard, what will 80 yards come to? Ans. $75.00.

8. What will 3 yards, 3 quarters and 3 nails of cassinet come to at ⅝ of a dollar per yard? Ans $2.46.

9. What will 65 yards, 2 quarters, and 1 nail come to at ⅞ of a dollar? Ans. $57.36¾.

10. What will 420 yards come to at $\frac{3}{16}$ of a dollar? Ans. $78.75.

11. What will 35 yards come to at $\frac{9}{16}$ of a dollar? Ans. $19.68¾.

12. What will 42 yards come to at $\frac{7}{16}$ of a dollar? Ans. $18.37½.

13. What will 63 yards and 3 quarters come to at $\frac{5}{16}$ of a dollar? Ans. $19.92¼.

14. Bought 200lbs. cheese at 9⅜c. pr. lb. " $18.75.

15. Bought 12 bushels, 2 pecks, 4 quarts of apples at 33⅓ cents per bushel? Ans. $4.21.

16. Bought 12 gallons, 3 quarts, 1 pint of brandy at $1.05 per gallon? Ans. $13.52.

17. Bought 12 lbs. 14 ozs. of tea, at $1.37½ cents per pound? Ans. $17.70.

18. Bought 25 lbs. 12 ozs. of nutmegs at $3.25 cents per pound? Ans. $83.68¾.

CUSTOM HOUSE CALCULATIONS.

To secure the exact collection of duties imposed on certain articles of imported merchandize, and on the tonnage of vessels employed in commerce, the law provides that the cargoes of vessels employed, in foreign commerce shall be inspected, and weighed or guaged by the custom house officers, and certain allowances made on boxes, casks, &c. containing goods on account of leakage, breakage, &c.

ALLOWANCES.

Draft, is an allowance, made from the allowance of each box, cask, &c. on account of probable waste.

Tare is an allowance made for the weight of the box, bag, cask, &c. containing the goods.

Gross weight is the whole weight, including the weight of the box, bag, or cask, containing the goods.

Neat weight is the weight of any parcel of goods after the draft and tare have been deducted.

DRAFT.

On a single box, &c. weighing 1 cwt. or 112 lbs.	1 lb.
On weighing above 1 cwt. and under 2 cwt.	2 lbs.
" " " 2 cwt. and under 3 cwt.	3 lbs.
" " " 3 cwt. and under 10 cwt.	4 lbs.
" " " 10 cwt. and under 18 cwt.	7 lbs.
" " " 18 cwt. and upwards,	9 lbs.

TARE.

Tare is computed on the remainder of any weight after the *draft* has been allowed.

On sugar in casks (except loaf) is	12 per cent.
On " in boxes, - -	15 "
On " in bags or mats, -	5 "
On cotton in bales, - -	2 "
On " in ceroons, -	6 "
On Glauber salts in casks, -	8 "
On nails in casks, - -	3 "
On pepper " - -	12 "
On " in bales, - -	5 "
On " in bags, - -	2 "
On sugar candy in boxes, -	10 "
On twine in casks, - -	12 "
On " bales, -	3 "
On cheese in hampers or baskets,	10 "
On " in boxes, - -	20 "
On candles " - -	8 "

On soap in boxes,	-	-	10 per cent.
On chocolate "	-	-	10 "
On shot in casks,	-	-	3 "

Tare on all other goods paying a specific duty is allowed according to the statement of the same in the invoice, which is considered the actual weight of the box, bag, cask, &c. The importer may have the invoice tare allowed, if he make his election at the time of making his entry, with the consent of the Collector and Naval officer.

Leakage allowed on all merchandize in casks on the guage, paying duty by the gallon, is 2 per cent.

Breakage, 10 per cent is allowed on all beer, ale, and porter in bottles, and 5 per cent, on all other liquors, or the importer may have the duties computed on the actual quantity at the time of entry. Common size bottles are estimated at the Customhouse to contain $2\frac{3}{4}$ gals. pr. doz.

Duties on foreign goods, are either *ad valorem* or specific.

Ad valorem duty is a certain per cent of the actual cost of the goods in the country from which they are brought.

Specific duty is fixed at a certain sum per ton, hundred weight, pound, square yard, &c.

Tare, *draft*, *&c.* are to be made, before the duties are computed.

1. Calculate the duty on an invoice of dry goods which cost in Liverpool $9,840 at 10 per ct. ad valorem?

Ans. $984.00.

2. Calculate the duty on 3 pipes of wine, allowance for leakage as in the table; duty $7\frac{1}{2}$ cents per gallon?

Ans. $27 78.

3. Calculate the duty on 10 gross of London porter, allowance for breakage as in the table; duty 20 cts. per gallon? Ans. $57.60.

4. Calculate the duty on an invoice of silk goods, which cost in Canton $6,400 at 10 per cent ad valorem?

Ans. $640.

5. Calculate the duty on 4 casks of Rochelle salts, invoiced at $10 per cwt. gross weight as follows: 1st cask, 1 cwt. 2 qrs. 12 lbs.; 2d, 1 cwt. 1 qr. 17 lbs.; 3d, 2 cwt. 3 qrs. 7 lbs; 4th, 4 cwt. 1 qr., draft as in table; tare 8 per cent, duty 15 per cent ad valorem? Ans. $14.09.

TARE AND TRET.

Tret is an allowance made on some particular articles on account of waste.

Tare is an allowance made by the seller to the buyer for the weight of the case, cask, box, bale, &c., in which goods are packed, calculated, at so much per cask, or at so much per cent, according to the nature of the goods.

Neat weight, is that quantity to be settled for, after all allowances have been deducted.

CASE I.

When the Tare is so much on a given quantity gross.

RULE.—Subtract the given tare from the given quantity, and the remainder will be the neat weight.

1. What is the neat weight of 1 hhd. of tobacco weighing 5 cwt. 2 qrs. 15 lbs. gross, when the tare is 3 qrs. 7 lb?

Gross,	5	2	15
Tare,	0	3	7
Neat wt.	4	3	8 Answer.

2. What is the neat weight of 8 hhds. of sugar, each weighing 7 cwt. 3 qrs. 20 lbs., tare in the whole 5 cwt. 1 qr. 19 lbs. Ans. 58 cwt. 0 qrs. 1 lb.

CASE II.

When the tare is so much per bag, box, hogshead, or other denomination.

RULE.—Multiply the given tare per bag, box, barrel, &c. by the number of bags, boxes, barrels, &c. and subtract the product from the gross, the remainder will be the neat weight.

1. How much is the neat weight of 25 kegs of raisins, each, gross 1 cwt. 2 grs. 15 lbs., tare 19 lbs. per hundred weight.

	cwt.	*qrs.*	*lbs.*	
	1	2	15	
				$5 \times 5 = 25.$
	8	0	19	
			5	
Gross weight,	40	3	11	
16 lbs. is ¼ =	5	3	9	
2 lbs. is ⅛	0	2	25	
1 lbs. is ¼	0	1	12	
19 = Tare,	6	3	18	

Neat weight—cwt. 33 3 21 Answer.

2. What is the neat weight and cost of 10 hhds. of tobacco, each weighing 5 cwt. 1 qr. 13 lbs. gross, and 16 lbs. tare per cwt. at $8.75 per. cwt.?

Ans. neat 46 cwt. cost $402.50.

CASE III.

When the tare is so much on the hundred weight.

RULE.—Divide the gross weight by the aliquot parts of a hundred weight, and deduct the amount of the result from the gross, and the remainder will be the neat weight. Case III. will be found applicable to the last examples.

CASE IV.

When the Tare and Tret are both allowed.

RULE.—First find the tare, which deduct from the gross, and the remainder will be the '*suttle*,' divide the suttle by 26,* the product will be the trett, which subtract from the suttle, the remainder will be the neat weight.

* Four pounds on the 104 lbs. is the usual allowance for tret; the reason we divide by 26 is, that 4 lbs. is 1-26 of 104.

1. There are 17 boxes of sugar, each 1 cwt. 3 qrs. 18 lbs. gross, tare 16 lb. per cwt., trett 4 lbs. per 104 lbs.; what is the neat weight, and what is the value at $7.60 pr. cwt?

cwt.	*qrs.*	*lbs.*						
1	3	18	× 1				$7.60	*qrs.* *lbs.*
			4 × 4 = 16				26	3 4
7	2	16					4560	
		4					1520	
30	2	8		*qr.*			19760	
1	3	18		2	½		380	
				1	¼		190	
32	1	26	16 lbs.	*lb.*			027	+
4	2	15						
				4	¼		$203.57	
27	3	11	suttle.					
1	0	7	Tret, 4lb. 1/26					
26	3	4	neat.					

2. What is the neat weight and value of 12 bags of coffee, each 2 cwt. 1 qr. 10 lbs. gross, tare 18 lbs. per cwt., tret 4 lb. per 104 lbs. at $19.60 per cwt.?

Ans. 22 cwt. 2 qrs. 18 lbs. cost $444.15.

CASE V.

When the tare is rated at so much per cent on the gross.

RULE.—Multiply the gross by the tare per cwt. and divide by 100 for the tare, then calculate the amount as before.

1. What is the value of 15 hhds. of loaf sugar gross weight 68 cwt. 3 qrs. 21 lbs., tare 12 per cent. at 14 cts. per pound? Ans. $951.22½.

2. What is the value of 4 hhds. of tobacco, weighing as follows:

	cwts.	*qrs.*	*lbs.*	
No. 1 =	6	3	18	Tare 12 lb. per 112 lbs. at $3.75 per cent.
" 2 =	7	0	10	
" 3 —	5	3	26	
" 4 —	8	0	03	
Gross weight,	28	0	01	Ans. $93.78.

INTEREST.

Q. What is the consideration or legal allowance for the use of money called?

A. It is called interest or premium.

Q. What is the sum lent called?

A. The principal.

Q. What is the legal interest or premium of $100 per annum called?

A. The rate per cent.

Q. Why is it called the rate per cent?

A. Because, for a loan of $100, six dollars is allowed for its use for one year, it being considered the legal per centage in every State in the Union, except New York, in that state, the rate is $7 per annum.

Time.—The number of years, months, or days for which interest is to be computed.

Amount.—The principal and interest added together is called the amount.

GENERAL RULES AND THEOREMS FOR CALCULATING SIMPLE INTEREST IN DOLLARS AND CENTS.

THEOREM I.

This theorem is deduced from the principle, that any sum, at simple interest will double itself in 16 years, 8 months, or 200 months.

Putting P = any given principal, T = any given time in months we have,

$$200 : T :: P : \frac{T}{2} \times \frac{P}{100} = \frac{TP}{200}$$ which expressed in words, reads thus:

RULE.—Multiply the given principal by half the given time in months, pointing off two figures on the right hand for decimals.

THEOREM II.

This theorem furnishes the shortest method possible for calculating simple interest for days: it is well known, that in banking institutions, interest is universally charged on all loans at the rate of one per cent. for sixty days. Therefore, it is manifest, that putting P = any given principle, and T = any given time in days less than a year, we shall have $60 : T :: 1 = \frac{T}{60}$ = the interest of $100 for the given time. And again,

$$100 : P :: \frac{T}{60} : \frac{T \times P}{6000} = \frac{T}{6} \times \frac{P}{1000} = \text{the}$$

interest of P for the whole time.

RULE.—Multiply the principal by *one-sixth* of the given number of days, and divide their product by 1000, for cents, or point off 3 decimal places on the right hand, and you have the interest required.

To find the interest of any sum for days.

RULE.—Multiply the principal by $\frac{1}{6}$ of the days which denote as so many thousandths for decimals, the product when pointed will be the interest.

By analysing, we find that as $100 gives $6 interest in 360 days, it will give a proportional interest for any other number of days, of which 360 is a multiple. Therefore, it will be clearly seen, that the interest of $100 for 60 days at 6 per cent. is $1.00; hence, it is manifest, that the interest for any given sum for 60 days, will be as many cents as there are dollars in the principal, for instance, the interest of 48 dollars for 60 days, is 48 cts. for $50 principal interest, 50 cents, and so on.

1. What is the interest of $80 for 60 days? Ans. 80c.
2. What is the interest of $90 for 60 days? " 90c.
3. What is the interest of $40 for 60 days? " 40c.
4. What is the interest of $45 for 60 days? " 45c.

CASE I.

Equimultiples of sixty days.

From the preceding illustration it is plain, that if the interest of 80 dollars for 60 days be 80 cents, the interest for 30 days will be 40 cents.

1. What is the interest of $240 for 120 days? Ans. $4.80.

Illustration.—The interest of $240 for 60 days would be 240 cents or $2.40, and as the ratio of 60 to 120 is as 1 to 2. Consequently, $2.40 × 2 = $4 80.

2. What is the interest of $300 for 120 days? Ans. $6.00.
3. What is the interest of 350 for 120 days? " 7.00.
4. What is the interest of 460 for 120 days? " 9.20.

METHOD OF CALCULATION IN BANKING INSTITUTIONS.

RULE.—Multiply the principal by the number of days, including the days of grace, and the day on which the money was paid to the drawer, and divide by 60.

1. What is the interest (or bank discount,) on a note of $100 for 60 days.

$$.60 + 4 = 64 \times 100 = \frac{6400}{60} = 106\tfrac{2}{3} \text{ or } \$1.07.$$

But the usual method of Bank calculation is more concise; thus, $100 for 60 days, agreeably to the analysis of Theorem 2, is $1.00.

3 days grace + 1 day for payment.

4 days being = to $\frac{1}{15}$ of 60 is equal to .07 nearly,

$1.07 as before.

2. What is the interest on a note of $100 for 90 days?

Ans. $1.57.

Interest of $100 for 60 days is - -	$1.00
100 for 30 days is - -	0.50
Interest for 3 days grace and day of payment, viz: 4 days is - - - - -	.07
	$1.57

3. What is the interest on a note of $100 for 116 days? Ans. $2.00.

Operation.—116 + 4 = 120 = 60 × 2.

The interest of $100 for 60 days is - $1.00
Multiply by - - - - 2

Answer, $2.00

ANOTHER METHOD.

RULE.—From the amount of the note, point of two figures to the right hand, this done, divide by 15, add the quotient to the principal, and the sum will be the interest required.

4. Calculate the interest on a note of $100 for 60 days?

Illustration. $\frac{1.00}{15}$ = .06⅔, which added to $1.00 = $1.00 + .06⅔ = $1.06⅔ in Bank, $1.07.

If for 90 days analyse it thus: for 60 ds. set down $1.07
for 30 ds. " 50

$1.57

5. Find the Bank interest on a note of $100 for 30 days? Ans. $0.57.

6. Find the Bank interest on a note of $240 for 90 days? Ans. $3.67.

Note.—When no per centage is named, 6 per cent. is understood.

CASE II.

When the time is one year, and the rate per cent. any number of dollars.

RULE.—If the principal be dollars, point out 2 places for cents. 2d. If it be in dollars and cents, point out 4 places for the decimal parts of a dollar. 3d. If in dollars, cents and mills, point out 5 places for the correct answer.

1. What is the amount of $144 for 1 year at 6 per cent? Ans. $152.64.

114
6

Interest, $8.64 We point out 2 places for cents
Principal, 144.00 agreeable to Rule 1st.

Amount, $152.64

2. What is the amount of $240.75 for one year at 6 per cent? Ans. $255.19½.

3. What is the interest of $640 for 1 year? Ans. $38.40.

4. What is the interest of $20.33, " " 1.22.

5. What is the interest of $1620 " " 97.20.

6. What is the interest of $19.64 for 2 years? Ans. $2.36 nearly.

To find the interest for years and months.

CASE III.

Bring the years to months, and multiply the principal by the number of months, and half the product will be the interest, or take half the months and multiply it by the principal.

1. What is the interest of $325 for 12 years? Ans. $234.00.

2. What is the interest of $617.56 for 25 years? Ans. $926.34.

3. What is the interest of $17,696.56 for 10 years? Ans. $10,617.93.

4. What is the interest of $250 for 11 years? Ans. $165.00.

5. What is the interest of $13.93¾ for 3 years? Ans. $2.50.

To find the interest for months at six per cent.

RULE.—Multiply the principal by half the number of months, and proceed as before.

1. Find the interest of $240.75 for 2 months at 6 per cent per annum? Ans. $2.40¾.

2. Find the interest of $320.25 for 3 months?
Ans. $4.80.3 +

3. Find the interest of $480.90 for 4 months?
Ans. $9.61.8.

4. Find the interest of $325.92 for 5 months?
Ans. $8.14.8.

5. What is the amount of $240.92 for 2 years, 10 months at 7 per cent.? Ans. $288.70.2.

6. What is the amount of $325.15 for 3 years, 5 months, at 5 per cent.? Ans. $380.70.

CASE IV.

When the rate of Interest is at any per centage more or less than six per cent.

RULE.—Find the Interest of the given PRINCIPAL at 6 per cent., as before directed; then multiply that interest by the given rate per cent. more or less than 6 per cent. and divide the product by 6, and you get the interest required.

1. Calculate the interest on a note of $275 for 4 months at 7 per cent.? Ans. $6.41§.

2. Calculate the interest of $350 for 5 months, at 5 per cent.? Ans. $7. 29 +

3. Calculate the interest of $248.75 for 11 months, at 7 per cent.? Ans. $15.96.

4. Calculate the interest of $365.67 for 7 months, at 3 per cent.? Ans. $6.40.

5. Calculate the interest of $600 for 15 months at 5 per cent.? Ans. $37.50.

6. Calculate the interest of $90.80 for 19 months, at 6 per cent. per annum? Ans. $8.62.6.

CASE V.

When the Interest is required for any number of years and months.

RULE.—Bring the years and months, to months, take half the sum; multiply that half sum, or half the number

of months, by the principal, for the interest at 6 per cent. according to Rule, 2nd case.

1. What is the interest of $65 for 3 years, 4 months, at 6 per cent. per annum? Ans. $13.00.

2. What is the interest of $199.11 for 3 years 8 months at 6 per cent? Ans. $43.80.4.

3. What is the interest of $98 for 4 years 2 months? Ans. $24.50.

4. What is the interest of $1298.40 for 4 years 8 months? Ans. $363.55.2.

CASE VI.

To find the Interest of any given principal for any number of years, months and days, at six per ct. per annum.

RULE.—Bring the years and months to months, take one-sixth of the days, which annex to half the months, multiply that number by the principal, and point out three decimal places in the product. One for mills and two for cents, which will give the correct answer.

It may be asked, why we take ⅙ of the days, an answer is at hand; because, 30 days is the general average of a month;—Days are therefore; thirtieths of a month, and sixtieths of half months, because the Interest of any sum at 6 per cent. is exactly ½ per cent. a month.—Hence if ⅙ of the days be taken they will become tenths, or decimals of a month.

1. What is the interest of $400 for 3 years, 4 months and 12 days, at 6 per cent. per annum? $80.80.

	yrs.	*m.*	*d.*
Illustration,	3	4	⅙)12
	12		—
			2
	½)40		
	202		
	400		

$80.80.0 Answer.

2. Calculate the interest of $360 for 2 years 8 months and 18 days. Ans. $58.68.

3. Calculate the interest of $800 for 1 year 8 months and 6 days? Ans. $80.80.

4. Calculate the interest of $240.65 for 4 years, 4 months and 15 days? Ans. $63.17.

5. Calculate the interest of $67.50 for 1 year, 7 months and 7 days? $6.49

6. Calculate the interest of $23.19 for 2 years, 5 months and 11 days? $3.40.5.

CASE VII.

When the Interest is required from a certain day of the month, in a year, to a particular day of the month in the same or in another year.

1. What is the interest of $30, from Feb. 6th, 1838, up to June 22d, 1840? Ans. $4.28.

2. What is the interest of $35.61 from Nov. 11th, 1831, up to Dec. 15th 1833? Ans. $4.47.4.

3. What is the interest of $11.10½, from April 17th, to Dec. 7th, 1832, at 7 per cent.? Ans. $0.49.6.

4. What is the interest of $369.29, from April 30th 1830, to July 31st 1832, at 4 per cent.? Ans. $33.28.

5. What is the interest of $1728.75, from Nov. 19, 1823, up to June 18th, 1826? Ans. $267.66.8.

6. What is the interest of $99.99.9, from Jan. 1st, 1800, to Feb'y 29th, 1832? Ans. $192.96.4.

PARTIAL PAYMENTS ON BILLS AT INTEREST.

In the Supreme Court of the United States, and, indeed, in most courts of the several States, the following rule is generally adopted, for estimating interest on Notes and Bonds; when partial payments have been made.

GENERAL RULE.

Compute the interest on the principal sum, from the time when the interest commenced, to the first time when a payment was made, add that interest to the principal,

and from the sum subtract the payment made at that time, together with the preceding payments, if any, and the remainder forms a new principal; on which compute and subtract the interest, as upon the first principal, and proceed in the same manner to the time of judgment.

Wilmington, Del. June 17, 1829.

For value received, I promise to pay Joseph Miller, or order, on demand, seven hundred and sixty-nine dollars, and eighty-seven cents, with interest.

$769.87. NELSON CLELAND.

Attest, ELI HILLIS.

On this note are the following payments: March 1, 1830, received seventy-five dollars and fifty cents. June 11, 1831, received one hundred and sixty-five dollars. September 15, 1831, received one hundred and seventy-one dollars. January 21, 1832, received forty-seven dollars and twenty-five cents. March 5, 1833, received twelve dollars and seventeen cents. December 6, 1833, received ninety-eight dollars. July 7, 1834, received one hundred and sixty-nine dollars. What remains due Sept. 25, 1835? Ans. $211.92.

METHOD OF OPERATION.

Principal carrying interest from the date of the note, June 17, 1829, to March 1, 1830.

	yrs.	*m.*	*d*		
	1830	3	1	Principal,	$769.87
	1829	6	17	Int. for 8m. 15d.	32.72
diff. of time.	0	8	15		$802.59
				Subtract first payment,	75.50
				Balance for new principal,	$727.09

	yrs.	*m.*	*d.*		
	1831	6	11		
	1830	3	1	Interest,	55.74
diff. of time.	1	3	10		782.83
				Subtract second payment,	165.00
				Balance for new principal,	$617.83

	yrs.	m.	d.		
	1831	9	15		
	1831	6	11	Int. for 3m. 4d.	9.68
diff. of time.	0	3	4		$627.51
				Subtract third payment,	171.00
				Balance for new principal,	$456.51
	yrs.	m.	d.		
	1832	1	21		
	1831	9	15	Int. for 3m. 6d.	7.30
diff. of time.	0	3	6		$463.81
				Subtract fourth payment,	47.25
				Balance for new principal,	$416.56
	yrs.	m.	d.		
	1833	3	5		$416.56
	1832	1	21	Int. for 1y. 1m. 15d.	28.12
diff. of time.	1	1	15		$444.68
				Subtract fifth payment,	12.17
				Balance for new principal,	$432.51
	yrs.	m.	d.		
	1833	12	6		
	1833	3	5	Int. for 9m. 1d.	19.54
diff. of time.	0	9	1		$452.05
				Subtract sixth payment,	98.00
				Balance for new principal,	$354.05
	yrs.	m.	d.		
	1834	7	7		
	1833	12	6	Int. for 0y. 7m. 1d.	12.45
diff. of time.	0	7	1		$366.50
				Subtract seventh payment,	169.00
				Balance for new principal,	$197.50
	yrs.	m.	d.		
	1835	9	25		
	1834	7	7		
diff. of time.	1	2	18	Int. for 1y. 2m. 18d. =	14.42
Due on the 25th September, 1835,					$211.92

Baltimore, March 10, 1840.

For value received I promise to pay Simon Kemp, fifty four dollars and eighteen cents, with interest.

$54.18. JAMES HOOPER.

Attest, JOHN E. STANSBURY.

On this note are the following payments: June 10, 1840, received twelve dollars and twenty-five cents. August 16, 1840, received ten dollars. October 21, 1840, received twenty dollars. March 4, 1841, received five dollars. What sum will be due on the 4th day of July, 1841? Ans. $9.04.

MISCELLANEOUS QUESTIONS IN INTEREST.

CASE I.

Principal interest and time given to find the rate per cent.

1. At what rate per cent. must $500 be put on interest to gain $120 in 4 years?

RULE.—Multiply the interest by the time, and subtract the product from the amount, the remainder will be the interest gained at 1 per cent. for the given time, provided the difference be less than the principal, divide the interest by that remainder, and the quotient will be the rate per cent. required.

Illustration.—$120 \times 4 = 480$, $500 - 480 = 20$, and $\frac{120}{20} = 6$ per cent as required.

Proof.—$500 at 6 per cent per annum gives $30 interest, which being multiplied by 4, = $120 the interest of $500 for 4 years.

2. If I receive $60 interest for the use of $600, 1 year, 8 months, what is the rate per cent. Ans. 6 pr. ct.

3. If I pay $200 for the use of $2000, for 2 years 6 months, what is the rate per cent? Ans. 4 per ct.

4. At what rate per cent. must $400 be put to interest to gain $120 in 5 years? Ans. 6 per ct.

5. At what rate per cent. must $500 be put on interest to gain $120, in 4 years? Ans. 6 per ct.

CASE II.

When interest multiplied by the time exceeds the principal.

1. At what rate per cent. must $125, be put to interest to gain $37.50 in 6 years. Ans. 5 per ct.

2. At what rate per cent. will $480 yield, $90 interest in 3 years, 1 month, and 15 days? Ans. 6 per ct.

3. If $225 gain $108 in 8 years, what is the rate per cent.? Ans. 6 per cent.

4. At what rate per cent. must $120 be on interest to amount to $133.20 in 16 months. Ans. 8¼ per cent.

5. At what rate per cent. must $280 be on interest to amount to $411.95 in 6½ years? Ans. 7¼ per cent.

CASE II.

1. Suppose $1000, at 4½ per cent per annum, amount to $1281.25. How long was it at interest?

Ans. 6 years 3 mos.

2. In what time will $1600 amount to $2048 at 4 per cent. per annum? Ans. 7 years.

CASE III.

1. In what time will the interest of $600 be equal to the principal at 6 per cent.? Ans. 16 years 8 mos.

GENERAL RULE.

Divide 100 by the given per centage, and the quotient will be the time in years; if there be a remainder, multiply it by 12, and divide by the rate per cent.

1. For months, thus,

```
6)100      4
   ---    12
          --
16 yrs. 6)48(8 months.
          ----
```

2. At 5 per cent., in what time will any sum of money double itself at interest? Ans. 20 years.

3. At 4 per cent. in what time will any sum of money double itself, at interest? Ans. 25 years.

4. At 6 per cent., in what time will any sum of money treble itself at interest? Ans. 50 years.

5. In what time will the interest of $240, at 6 per ct., be treble the principal? Ans. 50 years.

6. A certain property valued at $1500 rents for $132 annually, required the rate of interest? Ans. 8¼ per ct.

INSURANCE.

MARINE INSURANCE.

The subjects of *Marine* insurance are, ships, merchandise, freight, &c.

The following examples will clearly illustrate the principles of Marine Insurance; Real Estate, or Property Insurance.

1. Suppose Ezekiel Dorsey, of Baltimore, shipped on board the brig *Nimble*, Farrell, master; and consigned to David Dunham, Commission Merchant, Liverpool, to sell for his account:

120 bales cotton, cost	$6735.00
1000 barrels Flour at $5.00,	5000.00
Shipping expenses paid,	265.00
Amount of shipment and expenses,	$12000.00

Before the brig sails, Mr. Dorsey is anxious to have his property insured. Now admitting the rate of insurance to be 1¼ per cent. premium, and the cost of the policy $1.25 cts. How is the amount to get insured obtained, so as to cover all expenses accurately?

RULE.—As 100 less the rate of premium is to 100, so is the sum of cost, charges and policy to the amount required, to get insured

Cost and charges	$12000.00
Policy	1.25

100—1¼ = 98¾ : 100 : : 12001.25 : $12184.01

The amount to get insured, to cover all expenses; hence, Mr. Dorsey would have to pay to the insurance company $184 01.

REAL ESTATE.

2. My property in Baltimore is worth $30,000, for what amount must I get it insured, so as to cover cost and charges, insurance being 1 per cent. premium. Policy $1.25. Ans. $30,304.30—of which $304.30 is to be paid to the insurance company.

MERCHANDISE.

3. Effected insurance on my Warehouse and Merchandise therein, which cost me $18,000; what sum must I get it insured for, the insurance being 3 per cent., policy, $1.25. Ans. $18,557.99—of which the Underwriters receive $557.99.

CASE II.

When a Commission Merchant ships a cargo, to his correspondent, and therefore cost, charges, premium and policy, are all included.

RULE.—Add $\frac{1}{100}$ of the rate of commission to unity, multiply the sum by the rate of insurance, and call the product *b*, then as 100 less *b*: 100 more the rate of commission : : the sum of the cost, charges, and policy, to the amount sought, which is to be insured.

4. York and M'Allister, Commission Merchants, New Orleans, shipped on board the brig *Orleans*, Lewis, master, and consigned to Lewis Laroque, London, for his account, 920 bales of cotton, cost, $64,534.00

Paid shipping expenses, 266.00

Effected insurance of the invoice amount by the American Insurance Company, at 4 per cent., policy $1.25. Ans. the amount to get insured to cover all expenses, is $71,024.33. Amount of insurance at 4 per cent. is $2842.22

Commission on the whole, $67,642.22 at 5 per cent. is 3382.11

Proof $71,024.33

5. What sum must a policy be taken out for, to cover $2475, when the premium is 10 per ct. Ans. $2750.

6. What is the premium on $896, at 12 per cent.
Ans. $107.52.

7. A certain company own a cotton factory in Pittsburgh, valued at $26,250 for what sum must a policy be taken out, to cover cost and charges at 12½ per cent.?
Ans. $30,000.

COMMISSION.

1. What is the commission on $850 at 6 per cent.?
Ans. $51.

2. Calculate the commission on $37,702.46 at 5 per cent.? Ans. $1885.12.

3. The sales made by an Auctioneer amount to $209,723; what is his commission at 5 per cent.?
Ans. $10,486.15.

4. An Auctioneer's commissions at 5 per cent. on sundry sales in the city of New York, in 1839, amounted to $13,279.58, required the amount of sales made?
Ans. $265,591.60.

5. A merchant having $1728 in the Chesapeake Bank of Baltimore, wishes to withdraw 15 per cent.; how much will remain? Ans. $1468.80.

DISCOUNT

Is an allowance made on a bill, or any other debt not yet become due for prompt payment.

The discount taken from the principal leaves the present worth, or value of a bill, when discounted.

Q. By having the present worth, how is the discount obtained?

A. Subtract the present worth from the principal, and the remainder will be the discount.

RULE.—To find the present worth at 6 per cent., if the time be for years or months; as 100 *plus* half the months is to 100, so is the given sum to the present worth.

1. What is the present worth of $1333.20, due 1 year 10 months hence?

Illustration.

yrs.	*m.*				
1	10		100		
12			11		
2)22		as	111 —	100	133.20
11					100

dolls. cts.

111)1332000($120.00
111
222
222

2. What is the discount of $133.30, due 1 year, 10 months hence? Ans. $13.20.

Principal, $133.20
Present worth, 120.00

$13.20 discount as required.

3. What is the present worth of a note for $520, due 5 years hence? Ans. $400.

4. What is the discount on the above mentioned note, for $520, due 5 years hence? Ans. $120.

5. What is the present worth of $775.50 due in 4 years, at 5 per cent. per annum? Ans. $646.25.

6. What is the discount of $802.50 at 7 per cent. due one year hence? Ans. $52.50.

7. What is the discount on a note of $117.60, due 1 year hence, at 12 per cent? Ans. $12.60.

8. What is the present worth of $1350, due 5 years, 10 months hence? Ans. $1000.

BUYING AND SELLING STOCK.

1. What is the amount of $1564 United States Bank Stock at 114 per cent?

Operation.

$1564
114

6256
1564
1564

$1782.96 Answer.

2. Sold 15 shares $100 each, of the Marine Bank of Baltimore, at 13 per cent. advance, what is the amount?

Ans. $1695.

3. What is the value of 10 shares in the Philadelphia and Trenton Railroad stock, at 85 per cent. original shares being $100? Ans. $850.

4. What must be given for 8 shares in the Baltimore and Ohio Railroad stock at 10 per cent advance, the original cost being $100 each? Ans. $880.

5. What will $1686 corporation notes of the city of Wilmington, Del. be worth at 91½ pr. ct. Ans. $1542.69.

BARTER.

When one commodity is traded for another, it is called *Barter.*

1. How much sugar at 10 cents per lb. must be given in barter for 750 lbs. of raisins at 6 cents per lb.?

Ans. 450 lbs.

Illustration.—750 lbs. raisins at 6 cts. is $45. Now the question is, how many lbs. of sugar must we get for $45; divide $45 by 10, and we get 450 lbs.; hence, 450 lbs. sugar at 10 cts. is equal to 750 raisins at 6 cts. $450 \times 10 = 750 \times 6 = \$45$.

2. Aaron Abel, bought of Ben. Bailey, 6 hogsheads of rum, containing 410 gallons, at $1.17 per gallon, and

253 pounds of coffee at 21 cents per pound, in part of which he pays $21 in cash, and the balance in boards at $4 per thousand, how many feet of boards did the balance require? Ans. $127957\frac{1}{2}$ ft.

3. Richard Rich has 240 bushels of rye, which cost him 90 cents per bushel, this he trades with Peter Parley at 95 cents for wheat, that stands Parley 99 cents per bushel, how many bushels of wheat is he to receive in trade, and at what price is it to be rated in order to make the barter equal?

Ans. $218\frac{182}{209}$ bush. at $1.04\frac{1}{2}$ cts. per bush.

4. A. has sugar which he barters with B. for 4 cents per lb. more than it cost him against tea, which cost B. 40 cents per lb., but which he puts in barter at 50 cents, what did A's sugar cost him per lb. Ans. 16 cts.

5. A merchant delivered 3 hhds. of wine at $1.10 per gallon for 126 yards of cloth, what was the cloth per yard? Ans. $1.65.

6. A farmer traded 20 bbls. flour at $5.60 per bbl., for salt at $3.50 per barrel, how many barrels of salt must the farmer receive for his flour? Ans. 32 bbls.

7. How much corn at 45 cts. per bushel will pay for 33 yds. cassinet at $1.05 per yd? Ans 77 bushels.

8. D. Greves has 100 cords oak-wood for sale, worth $4 per cord cash, and is offered $4.25, payable on a credit of twelve months, which is the most advantageous, the cash or credit sale? Ans. the credit by 94 cents, allowing discount at 6 per cent. per annum.

9. A. gave B. 2 hhds. of brandy at 75 cents per gallon, for 56 yards of cloth, what was the cloth per yard? Ans. $1.68\frac{3}{4}$.

10. What quantity of tea at $1.30 per lb. must be given for 2,500 lbs. of rice at $4\frac{1}{2}$ cents per pound?

Ans. 86 lbs. 8 ozs.

11. A grocer had sugar at 8 cents per lb. for some of

which, B. gave 750 lbs. of tea at $1.08 per pound, how many lbs. of sugar must B. receive for his tea?

Ans. 90 cwt. 1 qr. 17 lbs.

12. John Doe, bought of Richard Roe, 104 tons of iron at $10 per ton, and is to pay as follows, viz: in cash $600, 240 lbs. sole leather at 33⅓ cts. per lb., and 10 loads of coal, each containing 12 bushels at 37½ pr. bush. 90 gallons of brandy at the rate of $75 per hhd., and the balance in coffee at 12½ cents per lb., how much coffee is Richard Roe to receive? Ans. 1662⅔.

13. Two persons A. and B. barter, A. has 17 cwt. of hams at 12½ cents per lb., B. has 1400 lbs. cheese at $10 per cwt.; which of them must receive money and how much? Ans. A. $113.

14. A man exchanged 40 bushels of salt at 87½ cents per bushel for 200 bushels of oats at 18¾ cts. per bushel, how much was the balance in his favor? Ans. $2.50.

15. A farmer sold a grocer 20 bushels of rye at 62½ cts., 200 lbs. cheese at 10 cts. per lb.; in payment he received 20 gals. of molasses at 25 cts. and the balance in cash, how much money did he receive? Ans. $27.50.

16. A farmer sold a grocer 15 barrels of apples, each bbl. containing 3 bushels, at 40 cts. per bushel, 30 bush. of corn at 90 cts. per bushel, 500 lbs. of cheese at 8 cts. per lb., 200 lbs. of butter at 15 cts. per lb., 20 bushels of onions at 75 cts. per bushel. In payment he received 4 barrels of Monongahela whiskey, each 31½ gallons at 33⅓ cents per gallon, 80 gallons of molasses at 35 cents per gal., 4 bags of Laguayra coffee, each containing 150 lbs. at 10 cts. per lb., what is the balance? Ans. 0.

17. A. has B's note for $535, payable in 14 months, and to redeem it for prompt payment, gives him 160 bushels of corn at 45 cts. per bushel, 122 do. wheat at $1 per bushel, 80 of rye at 90 cts. (market price,) and the balance in brick at $10 per thousand, how many thousand must A. deliver? Ans. 23 thousand.

LOSS AND GAIN,

Is one of the imaginary accounts in Double Entry Book Keeping, and so called in order to supply the want of real or personal titles in recording gains and losses, which could not with propriety be placed to real or personal accounts. It is by this Rule, that merchants discover the gain or loss in business, and the rate per cent. in buying or selling goods.

Q. What does the words *buy* or *bought*, *sell* or *sold* imply in this rule?

A. Buy or bought, means the first cost of the goods, sell or sold, the sales made.

Q. What does the first cost or invoice refer to?

A. The debtor side of a merchandise account.

Q. What does the items sold refer to?

A. The credit side of a merchandise account.

Q. How is a gain ascertained?

A. By taking the difference between the first cost and the sales.

Q. How is a loss ascertained?

A. When the amount sold is less than the first cost, take the difference, that difference will shew the loss.

CASE I.

First cost and a gain given to find the rate per cent.

RULE.—Take the difference between the first cost and the sales. Then say as the first cost is to the difference, so is 100 to the rate per cent.

Ex. 1. Bought coffee at 10 cts. per lb. and sold it for 12 cts., how much is the gain per ct.? Ans. 20 pr. ct.

Sold for 12
Bought for 10

As 10 : 2 gain : : 100 = 20 pr. ct. Ans.
5 : 1

CASE II.

Given the first cost and a loss to find the rate per cent.

RULE.—As the first cost is to the loss, so is 100 to the rate per cent.

Ex. 2. Bought sugar at 8 cts. per lb. (which was damaged,) and had to dispose of it at 5 cts. per lb., how much is the loss per cent? Ans. 37½ pr. ct.

Bought for 8 cents.
Sold for 5 "

As 8 : 3 cts. loss : : 100 = 37½ pr. ct. Ans.
2 25

CASE III.

To gain a certain per centage.

RULE.—As 10 per cent is $\frac{1}{10}$ of a hundred, add $\frac{1}{10}$ to the first cost. Thus, $\frac{1}{10}$ of $4.00 is 40 cts. which added, make $4.40, the selling price required; or, agreeably to the common method, say as 100 : 110 : : $4.00 = $4.40 as before. The first method is more concise, and therefore, the best.

1. Admitting I purchase cloth at $4 per yard, and wish to gain 10 per cent, how much is the selling price?

2. Bought calico at 20 cts. per yard, and want to gain 25 per cent, how much must be the selling price per yard? Ans. 25 cts.

CASE IV.

Selling price, gain and rate per cent given to find the first cost.

RULE.—Add the gain per cent. to 100, then say as 100 with the given per centage added, is to 100, so is the retail price to the first cost.

1. Suppose the retail price to be $5 per yard for cloth, and the gain 25 per cent, how much is the first cost?

Ans. $4.00 per yard.

CASE V.

When a certain loss per cent. has to be sustained on the first cost to find the retail price.

RULE.—Multiply the first cost by the given per centage, and divide by 100 for the amount of loss sustained, the amount subtracted from the first cost will give the result; or, as 100 : 84 : : 840 : $705.60 as required.

Suppose I purchase a lot of goods for $840, at auction, and owing to their being damaged, have to sustain a loss of 16 per cent below the first cost, how much will they be sold for? Ans. $705.60.

CASE VI.

To find the profit per pound, per yard, &c. when the first cost and sales of a certain number of yards, pounds, cwts. &c., are given.

RULE.—Find the whole gain, and divide it by the number of yards, pounds, or cwts., &c.

Ex. 5. Bought a chest of tea, containing 340 lbs. for $289, and sold it for $408, what was the profit on each pound? Ans. 35 cts. per lb.

CASE VII.

When there is a loss per cent. sustained by the sale and the first cost is required.

RULE.—Subtract the rate per cent. from 100, then say as 100 less the rate per cent. is to 100, so is the amount of the sale to the first cost.

Ex. 6. Sold a lot of dry goods for $540, and had to sustain a loss of 10 per cent., what did they cost me?

Ans. $600.

CASE VIII.

When you purchase goods and wish to gain a certain sum on the whole purchase, so as to be able to sell them at a certain per centage.

RULE.—Add the sum you wish to gain to the amount of the purchase, then divide that amount by the number of yards, pounds, and cwts. for the answer required.

Ex. 7. A merchant tailor bought 100 yards of cloth for $256, how much must he sell it per yard to gain $44 on the whole? Ans. $3 per yard.

1. Bought knives at 20 cents each, and sold them for 25 cents, how much is gained per cent.? Ans. 25 pr. ct.

2. Suppose I buy cloth for $5 per yard, and sell it for $6, how much is the gain per cent.? Ans. 20 pr. ct.

3. A store-keeper sold 100 yards of silk at $1.50 per yard, which cost him $1.25 per yard, how much did he gain by the sale? Ans. $20 on the whole.

4. George Brown, merchant, Baltimore, purchased the following bill of goods, viz: 10 pieces domestic muslin, each 29 yards at 10 cents per yard, and 16 pieces calico, each 27¼ yards at 20 cents per yard, how much is the retail price per yard on the muslin and calico, and the whole gain allowing a gain of 25 per cent. for the sale of the goods?

A. The retail price of the muslin is 12½ cts. per yard, and the calico 25 cts. per yard—whole gain $29.25.

5. Sold a yard of cloth for $1.55, by which was gained at the rate of 15 per cent., what would have been the gain per cent. if it was sold for $1.72? Ans $27.61 +

6. A. sold cloth at 84 cents per yard, and gained 10 per cent., should it be sold for $1.20, what would be the gain? Ans. $57½.

7. Richard Rich purchased of D. Draper, 18 pieces of broad cloth at $8 per piece, of which he sold 10 at $9.60 per piece, 5 pieces at $9 per piece, and the balance he wishes to dispose of, on terms to gain 12 per cent. on the whole, at what rate must the remainder be sold at per yard? Ans. $6.76.

8. Sold a lot of dry goods for $5.50, and gained 10 per cent., how much was the first cost? Ans. $5.00.

9. Bought at auction a hhd. of molasses for $50, and having found, that there was a leakage of 5 gallons, I sold

it immediately, at 10 per cent. loss, how much did I receive for it? Ans. $45.

10. Bought rum at $1.25 per gallon, and by accident, so much leaked out, that I am willing to lose 20 per cent., at what price must I sell it per gallon? Ans. $1.00.

11. Bought for cash $1740 worth of hardware, and sold the lot for $2000 on a credit of 3 months, what is the gain per cent. per annum? Ans. 59$\frac{3}{7}$.

12. Bought goods to the amount of $1400 cash, and sold them at 4 months credit for $1600, what is the gain per cent. per annum? Ans. 42$\frac{6}{7}$c.

13. Bought 3000 gallons of molasses at a credit of 2 months, for 30 cents per gallon, and sold it again at 35 cents per gallon, on a credit of 3 months, required the gain per cent? Ans. 64$\frac{2}{7}$c.

EXERCISE QUESTIONS.

1. I purchased 100 boxes of prunes at $2.10 each, and by selling them at $3.50 per cwt. the gain is 25 per cent, the weight of each box is required? Ans. 84 lbs.

2. If I sell linen for 75 cents per yard, and gain 12$\frac{1}{2}$ per cent., what would the prime cost of 30 pieces, each 33$\frac{1}{3}$ yards, stand me in the whole? Ans. $670.

3. Bought a quantity of leather at 22 cents per lb., how should I sell it to gain 10 per cent profit?
Ans. 24c. 2m.

4. Admitting that a yard of muslin was sold for 32 cents, which cost 37$\frac{1}{2}$ cents, what is the loss per cent.?
Ans. 14$\frac{2}{7}$.

5. Sold 500 penknives at 17 cents a piece, and by so doing lost 9 per cent, what is the loss on the whole?
Ans. $8.40 +

6 Bought a hhd. of wine for a certain sum, but 15 gals. having leaked out, I sell the remainder for $2.21.6$\frac{3}{4}$ per gal., and thereby lose 5 per cent. on the cost, what was the cost? Ans. $112.

COMPANY ACCOUNTS.

Partnership or Company transactions, (in an Arithmetical sense) is a Rule by which merchants in partnership adjust their accounts in proportion to stock and time.

CASE I.

The gain or loss, with the several sums at hazard given, to find each partners share.

RULE.—Multiply the whole gain by each man's fraction part of the stock—or, agreeably to the Old Method; say, as the whole stock is to the whole gain, so is each man's share of the stock, to each man's share of the gain.

1. Three men, A, B and C, entered into partnership for 2 years, with a capital of $6000, A put in $2500, B $1500 and C $2000, they gain $1080; required, each man's share of the gain?

Solution.—As the whole stock in trade is $6000, A's share would be $\frac{2500}{6000} = \frac{5}{12}$; B's $\frac{1500}{6000} = \frac{1}{4}$, and C's $\frac{2000}{6000} = \frac{1}{3}$. Again, $\frac{5}{12}$ of $1080 = $450, A's share, and $\frac{1}{4}$ of $1080 = $270, B's share, and C's $\frac{1}{3}$ of $1080 = $360.

Proof.—A's share of gain is			$450.00
B's	do.	do.	270.00
C's	do.	do.	360.00
		Whole gain	$1080.00

2. A and B enter into partnership; A has in goods at cash price $3400 worth, and cash $1300: B puts in $1200, and agrees to pay for A, a debt of $1100, for which A gives B a *title* to that amount of his goods. Now suppose A, agrees to take B's note for what B's funds want, of being equal to his own, (say the note bears legal interest, and is not reckoned in the partnership,)

what amount should the note be drawn for, to make them equal?

Solution.—Amount of A's goods $3400.00
" Cash 1300.00

4700.00
A sells to B, goods for 1100.00

3600.00
B's cash $1200.00
Goods 1100.00 — 2300.00

$1300.00 difference.

A's stock in trade then would be $3600—650 = $2950. A and B's equivalent composed of

Goods $1100.00
Cash 1200.00
Note 650 00

$2950.00 proof.

CASE II.

Apportioning the effects of a Bankrupt.

In apportioning the effects of a Bankrupt amongst his creditors, it is more convenient to find the proportion for one dollar, &c. Which will be a constant multiplier for each debt.

1. A Bankrupt owes $5000, his effects sold at auction, amounting to $4000, what will his creditors receive on a dollar? Ans. 80 cents.

2. A merchant having sustained many losses is obliged to become a bankrupt, his effects are valued at $1728, with which he can pay only 15 cents on the dollar, what did he owe? Ans. $11520.

3. A, B and C, freighted a ship with 108 tuns of wine, of which A had 48 tuns, B 36, and C 24, but by reason of stormy weather were obliged to cast 45 tuns overboard; how much must each man sustain of the loss?

Ans. A 20 tuns, B 15 and C 10.

4. Three merchants trading together lost goods to the value of $1860. A's stock was $2280, B's $11520 and C's $4800; what share of the loss must each man sustain?

Ans. A $288, B $1152, and C $480.

5. A ship valued at $25200 was lost at sea, of which $\frac{1}{3}$ belonged to A, $\frac{1}{4}$ to B, and the remainder to C; what is the loss on $1.00, and how much will each man sustain, supposing the owners effected an insurance of $18000?

A's $2400, B's $3600, and C's 1200.

The pro-rata share on a dollar is $\frac{1}{6}$.

CASE III.

When stocks have been put in trade for different periods of time, and settled with regard, both to stock and time.

RULE.—Multiply each man's stock and time, and then as the aggregate of products is to the whole gain, so is each man's stock to each man's share of the gain.

1. A, B, and C, join in company: A's stock is $100, for 12 months, B's 120 yards of cloth, for 8 months, and C's 240 bushels of wheat, for 7 months; they gained $1612, of which A had $400, B $512, and C $700; what was the value of B's cloth per yard, and C's wheat per bushel. Ans. B's cloth $1.60 pr yd, and C's wheat $1.25 pr bush.

2. A, B, and C, enter into partnership with a capital of $1100, of which A put in $250, B put in $300, and C $550; they lost by trading, 5 per cent. on their capital, what was each man's share?

Ans. A's loss $12 50, B's $15, and C's loss $27.50.

In *company accounts*, when the times and payments are equal, the shares of gain or loss are evidently in proportion to their respective stock—and when the stocks are equal, the shares are in proportion to the times of payment. But when stocks and times are unequal, the shares are in proportion to the products of stock and time.

This may be clearly demonstrated thus:

Suppose $100 in trade 12 months, gain $20; $50 in trade in 6 months, will gain $5, and both together $25;

for, as $100 \times 12 : 50 \times 6 : : 20 : 5$ and $20 + 5$; again, by composition $100 \times 12 + 50 \times 6 : 100 \times 12 : : 25 : 20$; gain of $100 in 12 months, and $100 \times 12 + 50 \times 6 : 50 \times 6 : : 25 : 5$, gain of $50 in 6 months, from which the truth of the rule is evident.

3. A, B and C having traded together, gain $126.80—what is each man's share, allowing that A put in $50 for 4 months, B $100 for 6 months, and C $150 for 8 months?

Ans. A $12.68, B $38.04, and C $76.08.

EQUATION OF PAYMENTS

Is a rule, for finding when any number of notes or bonds due at different times may be all paid at once, without loss to debtor or creditor.

RULE.—Multiply each payment by its time, divide the sum of the products, thence arising by the sum of all the payments, and the quotient will be the equated time required.

1. A. owes B. a bond for $100, due 2 months hence, and one for $500 due 18 months hence, what would be the equated time for paying them at once?

Operation.

	months.		
$100 ×	2	=	200
500 ×	18	=	9000
6)00		6)9200(	= 15⅓ *mo.* Ans.

2. C. owes D. $550, of which $100 is to be paid at three months, $200 at 5 months, and $250, in 8 months, but have agreed to make one payment of the whole, at what time must it be paid? Ans. 6 mos.

3. A man has owing to him $500 to be paid as follows, viz: $250 at 6 months, and $250 at 8 months, but it is agreed that the whole should be paid at one time, when must it be paid? Ans. 7 mos.

4. A. owes B. 5 bonds, for $945 each, payable at 3, 9, 11, 19, and 29 months, what time might they be all paid at once? Ans. 14⅓ mos.

AVERAGE TIME OF SALES.

CASE I.

1. Sold merchandise at sundry times, and on different terms of credit, as per statement annexed.

1840 January 1st, $1500 at 3 months, due 1st April.
February 10th, 250 at 2 " " 10th "
March 19th, 643 at 4 " " 19th July.
Sept'ber 1st. 1400 at 6 " " 1st March.

Required the average time of payment.

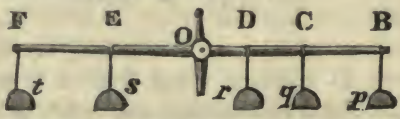
Ans. 21st August.

Statement of the preceding question.

		days.
Due 1st April	$1500 ×	0
" 10th "	250 ×	9
" 19th July	643 ×	109
" 1st March	1400 ×	334
	3793	3793)539937(142

142 days from the 1st day of April which will make the average time fall on the 21st day of August.

THEOREM *and General Rule, to find the average time that several bills of different dates, or different terms of credit, or both, become due.*



In the above diagram let p, q, r, s, t, &c., be the several payments to be made, and B, C, D, E, F, &c., the different periods of time at which those payments are to be made, and O, the average point of time, then it is manifest (on the principal of Simple Interest, that $p \times BO + q \times CO + r \times DO = t \times FO + s \times EO$, and putting $BO = x$, we have $p\,x + q\,(x - BC) + r\,(x - BD) = s\,(BE - x) + t\,(BF - x)$ by transposition

$px + qx + rx + sx + tx = q \times BC + r \times BD + s \times BE + t \times BF$ $(p + q + r + s + t)\ x = x = \frac{q \times BC + r \times BD + s \times BE + t \times BF.}{p + q + r + s + t.}$

Hence the following general rule: multiply the several payments to be made by the respective times from the first payment, add them together and divide that sum by the whole amount of the bills for the time sought, which is to be counted from the time on which the first payment falls due.

SOLUTION OF QUESTION I. CASE I.

334 1st March x 19th July. 10th April. 1st Ap.

Let x = average time from 1st April, then $1500\ x + 250\ (x - 9) + 643\ (x - 109) = 1400\ (334 - x)$ then $1500\ x + 250\ x - 2250 + 643\ x - 70087 = 467600 - 1400\ x$. Again, $1500\ x + 250\ x + 643\ x + 1400\ x = 467600 + 70087 + 2250$ and $x = \frac{2250 + 70087 + 467600}{3793} = 142$ days after the 1st day of April, which agrees with the 21st day of August, 1840, as before.

SOLUTION OF CASE I. BY ANOTHER PRACTICAL METHOD.

$\frac{1}{10}$ of the first bill is	$150	×	0	=	000	
"	2d	"	25	×	9	=	225	
"	3d	"	64	×	109	=	6976	
"	4th	"	140	×	334	=	46760	
			379				379)53961(142	

Note.—Agreeably to mercantile usage, a fraction less than one-half in dollars or days is omitted in the equation of payments, and when more than a half, it is considered as a unit.

CASE II.

1. Sold merchandise at different times, and on various terms of credit.

1839 September	6th	$100	for	1	month,	due Oct. 6th.
"	14th	125	"	1	"	" " 14th.
October	10th	175	"	2	"	" Dec. 10th.
November	14th	340	"	3	"	" Feb. 14th.
January	14th	456	"	5	"	" June 14th.

FIRST METHOD.

Solution.			*days.*
October	6th	$100 ×	0
"	14th	125 ×	8
Dec'ber	10th	175 ×	65
Feb'ry	14th	340 ×	131
June	14th	456 ×	251
		1196	1196)171371(= 144 +

144 days from the 6th October will come to February 27, 1840, at which time a note for $1196 would be due.

SECOND METHOD.

October	6th	$100 ×	251
"	14th	125 ×	243
Dec'ber	10th	175 ×	186
Feb'ry	14th	340 ×	120
June	14th	456 ×	000
		$1196	1196)128825(= 108 +

Due by average 108 days earlier than the 14th day of June 1840, which will be the 27th day of Feb. as above.

THIRD METHOD.

1839 September	6th	$100 ×	30
"	14th	125 ×	38
October	10th	175 ×	95
November	14th	340 ×	161
January	14th	456 ×	281
		1196	1196)207251(174 +

174 days from the 6th September, which will make the average time fall on the 27th day of Feb. 1840, as before.

AVERAGE CALCULATIONS BY INTEREST.

				days.	
September	6th	$100	×	30	at interest
"	14th	125	×	38	"
October	10th	175	×	95	"
November	14th	340	×	161	"
January	14th	456	×	281	"
		1196			Interest, $34.57

As $11.96 : 60 : : $34.57 = 174 days, nearly being the time required for $1196 to gain $34.57, the whole time by equation from the 6th Sept. to the 27th February.

CASE III.

To average personal accounts when goods are received in barter.

RULE.—Find the average time for the debtor and creditor sides of the accounts, and multiply the least sum by the difference of time, divide the product by the difference of the accounts, and the quotient will be the number of days to be carried forward.

Illustration.—Sold Barton & Co. Philadelphia, merchandise at different dates and credits for $600 due by average 6th September.

Received from Barton & Co. at different times, goods amounting to $1000, due by average on the 6th January, 1841, required the time of payment for the balance due the firm?

	From $1000
Least sum $600.	Take 600
	400

Difference of time from 6th September to 6th January.

600 × 91

400)546(00

136½ days.

Due by average 136¼ days from the 6th day of January, which falls due on the 23d day of May.

Dr. Barton & Co. Phila.		Barton & Co. Cr.	
1840.		1841.	
Sept. 6th to Mdse.			
As per average,	$600.00	Jan. 6th, by Mdse.	$1000.00
May 23d, balance,	400.00		
When due,	$1000.00		1000.00

From the 6th September to the 23d May (average time) is 227½ days, and $600 \times 227\frac{1}{2} = 1000 \times 136\frac{1}{2} = {}^{136500}_{6} = \$22.75$ equated interest for Dr. and Cr.

CASE V.

ANOTHER METHOD.

1. A. B. purchased merchandise on credit at different times, and is desirous to pay the whole amount at a certain period, required the time by equation?

Sales made as per bill rendered.

		dolls.	*days.*	*product.*
1840 May 4th	$52.00		× 4	
" 8th	13.87	66 ×	29	
June 6th	104.20	170 ×	4	
" 10th	84.16	254 ×	34	
July 14th	125.04	000 ×	0	
				379)11438(30

Thirty days counted back from July 14th will give the equated time on the 14th of June.

EXCHANGE.

Q. What is Exchange?

A. It is the paying or receiving money in one country, for its equivalent in the money of another country by means of Bills of Exchange.

Q. What is meant by a Bill of Exchange?

A. A Bill of Exchange is a written order for the payment of money at an appointed time.

Q. Who is the drawer?

A. The "maker and seller" of the bill.

Q. Who is the acceptor?

A. The person to whom the bill is addressed, is called the acceptor; when he engages, to pay the bill.

Q. What is meant by endorsing a Bill?

A. When the holder of a bill, disposes of it, he writes his name on the back, which is called endorsing.

Q. Are there other persons occasionally concerned in a Bill of Exchange?

A. Yes, the buyer or remitter, the seller or negociator, and the holder or possessor.

Q. Who should be the first endorser to a Bill?

A. The payee.

Q. What is a draft called, when a Merchant in the U. S. draws on his Banker in London?

A. "Bill on London," and *vice-versa*.

Drawee.—The person to whom a bill is addressed.

Payee.—The person to whom a bill is ordered to be paid.

Indorsee.—The person to whom a bill is made payable.

These are technical terms in law.

A Table of Foreign Gold and Silver Coins which pass current in the United States of America, with their Federal value, as established by Congress in 1834.

GOLD.	dolls. cts. m.	GOLD.	
Double Joannes, value in the United States,	$32.00 0	An English Guinea or Sovereign, pursuant to act of Congress,	$4.80
Single do.	16.00 0	French Pistole,	3.77
Half do.	8.00 0	French Guinea,	4.60
Quarter do.	4.00 0	French Crown,	1.10
Eighth do.	2.00 0	Dollar of Spain, Sweden or Denmark,	1.00
Testoon, or one sixteenth,	1.00 0	An English Shilling,	0.22
Moidore,	6.00 0	Spanish Pistareen,	0.20
Half do.	3.00 0		
Quarter do.	1.50 0		
Doubloon	14.93 $\frac{3}{4}$		

A Table of European Coins reduced to Dollars and Cents.

FRANCE.	cts. m.
Denier, - -	$0.00.08
Sol, or 12 deniers,	0.00.92
Livre Turnois or 20 sols.	18.52
Ecu or Crown 6 Livres,	1.10.00
Pistole 10 livres,	1.85.17
Louis d'or - -	4.44.40
Franc, - -	0.18.74
Five Francs, -	0.93.70

HOLLAND.	
Stiver, - -	$0.01.94
Scalin, -	11.64
Guilder or Florin, -	38.80
Rix Dollar, -	97.00
Ducat, -	2.07.86
Gold ducat, -	8.00.00

SPAIN.	cts. m.
Maravedie,	$0.00.30
Rial, - -	0.10.
Pistarine,	0.20.
Dollar, -	1.00.
Ducat of ex.	1.10.18
Pistole, -	3.60.
Pistarine of ex.	0.80.

DENMARK.	
Skilling,	$0.01.94
Duggen,	0.06.24
Marc, -	0.16.66
Rix Marc, -	0.20.83
Rix Ort, -	0.25.
Crown, -	0.66.66
Rix dollar, -	1.00.
Ducat, -	8.83.34

ITALY.	cts. m.
Soldi, -	$0.00.80
Chevelet, -	0.03.18
Livre, -	0.15.92
Testoon, -	0.23.88
Croisade, -	0.79.60
Perro of ex.	0.92.60
Genouine, -	1.36.12
Pistole, -	3.60.00

SWEDEN.	
Stiver, -	$0.00.72
Copper Marc,	0.02.88
Silver Marc,	0.08.64
Copper dollar,	0.11.52
Caroline, -	0.25.92
Rix dollar, -	1.03.70
Ducat, -	2.07.40

PORTUGAL.	cts. m.
Rea, - -	$0.00.12
Vintin, -	0.02.50
Testoon, -	0.12.50
Crusade of ex.	0.50.00
Milrea, -	1.25.00
Moidore, -	6.00.00
Joannes, -	8.00.00

SWITZERLAND.	
Fenning, -	$0.00.24
Cruitzer, -	0.00.92
Sol, -	0.02.77
Gulden, -	0.55.55
Rix dollar, -	1.00.00

FORM OF A BILL OF EXCHANGE.

Baltimore, June 16th, 1840.

Exchange for 8059 fr. 60 cts.

At usance, pay this first of Exchange (second of the tenor and date unpaid) to Peter Legrand, or order, Eight Thousand and fifty-nine Francs, sixty centimes, value received, and charge the same to account, with or without advice.

John Prentiss.

To Mr. Jean Barbeau, Bourdeaux.

Required the value of the above bill in United States currency? Ans. $1542.50 nearly.

NOTE AT BANK.

Baltimore, February 25th, 1840.

$500

Ninety days after date I promise to pay David Ring, or order, at the Marine Bank, Five Hundred Dollars, for value received.

John M'Nevin.

CHECK ON A BANK.

No. 1.

$150

Cashier of the Chesapeake Bank of Baltimore, pay to Richard Kelso, or bearer, One Hundred and Fifty Dollars, which charge to account of

George R. Richardson.

To J. Pinckney, Cashier.

PROMISSORY NOTE.

BY TWO OR MORE PERSONS.

On demand, we jointly and severally, promise to pay, Robert Hill, or order, with interest, at 6 per cent., Seven Hundred Dollars, for value received.

Baltimore, March 21st, 1840.

William P. Preston,
Chas. H. Pitts.

$700

NOTE PAYABLE TO BEARER.

$50

Six months after date, I promise to pay John Bennett, or bearer, Fifty Dollars, for value received. Baltimore, June 16th, 1840.

Albert Adams.

AN ORDER FOR MONEY.

Mr. E. Dorsey, Pay William Kemp, Ten Dollars and this shall be your receipt for the same.

Baltimore, July 10*th*, 1840. *James Thompson.*

AN ORDER FOR GOODS.

Baltimore, July 16*th*, 1840. *Mr. John Bennett,*

Pay the bearer Fifteen Dollars in Merchandize from your store, and charge, yours Respectfully, *William Wilson.*

A FOREIGN BILL OF EXCHANGE.

Baltimore, May 1st, 1840.

Exchange for £1800 Sterling.

At thirty days sight of this my second of exchange, (first and third of same tenor and date not paid,) pay to Fielding Lucas, Jr. One Thousand Eight Hundred Pounds Sterling, which place to account of *John Allen.*

To Mr. Benjamin Burton, London.

AN INLAND BILL.

Philadelphia, May 1st, 1840.

$500

Thirty days after date, pay to Joseph R. Chandler, or bearer, Five Hundred Dollars, value received.

Alfred J. Betton.

To Mr. John Dalzell, Merchant, Philadelphia.

LETTER OF CREDIT.

Messrs. Baird & Alexander:

Gentlemen: Allow me to introduce to your firm the bearer, Emelius Adams, a gentleman about commencing business. Should he make a selection from your stock to the amount of Five Thousand Dollars, I will be answerable for that sum, in case of his non-payment. I remain with much respect,

Yours, &c. *John Bennett.*

RECEIPT, GENERAL FORM.

$2425

Baltimore, January 22d, 1841.

Received of B. H. Richardson, Two Thousand Four Hundred and Twenty-five Dollars, in full, for balance of account. *Samuel Kingston.*

BILL OF LADING.

Shipped, in good order and well conditioned, by W. Wilson & Sons, on board the Barque John A. Robb, whereof Robert Walker is master, now lying in the Port of Baltimore, and bound for Liverpool, to say

being marked and numbered, as in the margin, and are to be delivered in the like order and condition, at the port of Liverpool, the dangers of the seas only excepted, unto Lewis Watson or to his assigns, paying freight for the said with primage and average accustomed.

In Witness whereof, the Master of the said vessel hath affirmed to Bills of lading, all of this tenor and date, one of which being accomplished, the others to stand void. Dated in the City of Baltimore on the 25th day of January, 1841.

Robert Walker.

NEGOTIABLE NOTE.

$800

Baltimore, January 26th, 1841.

Sixty days after date, I promise to pay to the order of Charles Keenan, Eight Hundred Dollars, without defalcation, for value received.

William H. Watson.

CASE I.

When Sterling money is required.

1. Suppose A. Allen of Baltimore, has occasion to draw on B. Burton of London, for $5462.50 when the exchange is at 9¼ per cent. (in favor of England) it is required to find how much sterling money the bill must be drawn for? Ans. $5000.

Example.—As 109.25 : 100 : : $5462.50 : $5000.

To reduce $5000 American money to sterling, multiply the dollars by 9, and divide by 40 (*i. e.*) $\frac{5000\times9}{40}$= £1125, the amount required.

Q. Why do we multiply the dollars by 9 and divide by 40?

A. Because, when the dollar is at par, 4s. 6d. make a dollar, and as 9 sixpences make a dollar, and 40 a pound sterling, therefore we multiply the dollars by 9 and divide by 40.

CASE II.

To change Pounds Sterling to Dollars.

RULE.—Multiply the pounds by 40, and divide by 9, and we get the answer in dollars—then as 100 London : 109.25 Philad. : : 5000 Lon. : $5462.50, the amount sought in federal money.

1. A merchant in Philadelphia has occasion to draw on Paris for the above sum of $5462.50, when the exchange with France is at 5 francs, 22½ centimes per dollar;—required, how much French money the bill must be drawn for? Ans. 28541.56¼ francs.

CASE III.

To change 5 Franc Pieces (French money) to Dollars.

RULE.—Say as the value of the 5 franc, agreeable to the rate of Exchange in French money is to $1.00 American, so is the amount of the bill in French money to its value in American currency.

3. What is the value of £948 16s. 8d. sterling in American currency?

Ex. As the value of a dollar in Great Britain, may be either below or above par, that is, more or less than 4s. 6d. the proper method is, to bring the pounds, shillings and pence to pence, and divide by the pence in a dollar, for the amount required, thus suppose the dollar to be at par,

	£	s.	d.
then	948.	16.	8
		20	
s.	d.		
4	6	18976	
12		12	
54		227720	($4217.04 nearly. Answer.
		216	
		117	
		108	
		92	
		54	
		380	
		378	
		200	*Note.*—Two ciphers are added
		216	in order to find cents.

4. How many milreas of Portugal are equal to $12500 American currency? Ans. 10,000 milreas.

5. In 10,000 guilders of Holland, how many dollars? Ans. $3880 U. S. currency.

6. In 10,500 crowns of Denmark, how many dollars? Ans. $7000.

7. In 400 livres turnois (or 20 sols) of France, how many dollars? Ans. $7408.

8. In $8000, how many rials of Spain? Ans. 80,000 rials.

9. In $6,000 U. S. currency, how many testoons of Portugal? Ans. 48,000 testoons.

10. How many pistoles of Spain are in $36,000?
Ans. 10,000 pistoles.

11. A of Amsterdam is indebted to B of Baltimore 3000 guilders, what is the amount, at 38 cts 8 m. per guilder?
Ans. $1164.

12. How many dollars will purchase a draft of 2500 Italian pistoles? Ans. $8000.

EXCHANGE TABLE FOR BUYING AND SELLING BILLS OF EXCHANGE, ABOVE AND BELOW PAR.

Rate per cent.	*Amount of* 1 £ or $1.	*Rate per cent.*	*Amount of* 1 £ or $1.	*Rate per cent.*	*Amount of* 1 £ or $1.
¼	1.0025	4	1.04	8¼	1.0825
½	1.005	4½	1.045	8½	1.085
¾	1.0075	5	1.05	8¾	1.0875
1.	1.01	5½	1.055	9	1.09
1¼	1.0125	5¾	1.0575	9¼	1.0925
1½	1.015	6	1.06	9½	1.095
1¾	1.0175	6½	1.065	9¾	1.0975
2.	1.02	6¾	1.0675	10	1.10
2¼	1.0225	7	1.07	10¼	1.1025
2½	1.025	7¼	1.0725	10½	1.105
2¾	1.0275	7½	1.075	10¾	1.1075
3.	1.03	7¾	1.0775	11	1.11
3½	1.035	8	1.08	11¼	1.1125

This Table is convenient for finding the amount of bills at any rate per cent. from ¼ to 11¼ per cent., above or below par.

CASE IV.

To find the amount of a Bill of Exchange when the rate per cent. is above par.

1. What is the amount of a bill of exchange for $750 at 4 per cent. above par? Ans. $780.

Illustration. Opposite to 4 per cent. in the annexed table, the tabular number is $1.04 \times 750 = \$780$.

2. What is the value of a bill of exchange for $1175 at $9\frac{3}{4}$ per cent. above par? Ans. $1289.56\frac{1}{4}$.

3. What is the value of a bill of exchange for $5333 at 7 per cent. above par? Ans. $5706.31.

CASE V.

To find the amount of a Bill of Exchange when the rate per cent. is below par.

1. What is the value of a bill of exchange drawn for $9630 at 7 per cent. below par? Ans. $9000.

RULE.—Divide the given sum, for which the bill is drawn for, by the *tabular number*, opposite the rate per cent., thus $\frac{9630}{1.07} = 9000$

2. What is the value of a bill drawn for $530 at 6 per cent. below par? Ans. $500.

3. James Nevins, Stock and Exchange Broker, (Exchange Place, Philada.) sold the following drafts.

No. 1, $450 at 6 per cent. above par,
2, 900 " "
3, 750 " "

Required the amount? Ans. $2226.

TABLE SECOND.

Shewing the Decimal value of the parts of a Pound Sterling, from 1d. to 19 Shillings.

d.	£	sh.	£
¼	0.00104	4	0.20
½	0.00208	5	0.25
¾	0.00312	6	0.30
1d.	0.00416	7	0.35
2	0.00832	8	0.40
3	0.0125	9	0.45
4	.01666	10	0.50
5	0.0208	11	0.55
6	0.025	12	0.60
7	.02916	13	0.65
8	0.333	14	0.70
9	.0375	15	0.75
10	0.04166	16	0.80
11	0.04583	17	0.85
1 sh.	0.0500	18	0.90
2	0.1	19	0.95
3	0.15		

Emory and Company, Exchange Brokers, Baltimore, sold the following drafts.

No. 1, $1500 at 3 per cent. below par
2, 2000 " 2 per cent. " "
3, 1875 " 2½ " " "

Ans. $5246.36 +

CASE VI.

Negotiating Bills.

1. P in Philadelphia owes L of Liverpool £349 19s. 3½d. sterling, to pay which he buys a bill at 2½ per cent. below par; what must he pay in United States currency?

Example shewing the use of Table 2d.

First put down		£349.95
For	3d	" 0125
For	½d	" 00208
		£349,96458 this reduced to

dollars by case 2nd × 40

9)139,98.58320

$1555.39813 = $1555.40 nearly

and 1555.40

1.025 = $1517.46 + Answer.

2. A Factor in Philadelphia, owes a merchant in Dublin £1500 sterling, to pay which he buys a bill at 4 per cent. above par; how many dollars did that bill cost him?

Ans. $6933⅓.

Let these examples be proved, by reducing them to sterling money.

3. A merchant in Bordeaux owes a merchant in Philadelphia, the nett proceeds of a consignment, amounting to $750.16, how many francs must he draw for, if exchange be at 19 cents per franc? Ans. 3948 frs. 21 cts.

4. What must be paid in New York for an invoice of goods, charged at 591 florins 17 stivers, allowing the exchange at 40 cents per florin, or 2 cents per stiver, and advancing on the invoice 60 per cent.? Ans. $378.78+

CIRCULAR EXCHANGE.

CASE VII.

1. London was ordered to remit to Paris 1000 crowns at 32d. sterling per crown, and to draw for the value upon Amsterdam, at 36s. 6d. Flemish per pound sterling; but when the order came up, bills on Paris were at 32¼d. sterling per crown, what must be the rate of exchange with Amsterdam to compensate the advance on the remittance? Ans. 36s. 2⅘d.

2. If the exchange in Hamburgh on London, at 2 usance* be 33, what should it be at sight, reckoning 1 per cent. for the time? Ans. 33s. 4d.

3. Suppose L of London has orders from P of Paris to remit to him at 23 livres 12 sous, (20 sous being equal to one livre) and to draw for the amount on A of Leghorn, at the exchange of 53d per dollar, but L finds the exchange of London on Paris is 24 livres—At what rate of exchange should he draw on Leghorn to fulfil the order? Ans. 52$\frac{7}{10}$.

4. A merchant in Baltimore shipped a quantity of flour which, when disposed of, amounted to 1040 milreas 500 reas, and received in return 17 pipes of wine, how much was the wine per pipe? Ans. $80.

INVOLUTON,

OR THE METHOD OF RAISING POWERS.

1. The first power of any number is unity multiplied by such number.

2. The second power of a number is found by multiplying any number by itself.

---

*The term *usance* is French, and signifies the usage of different countries, in relation to the payment of bills: usances vary from 14 days to 1, 2 and 3 months after the date of a bill.

EXAMPLES.

Let 2 be raised to all the successive powers, from the 2d to the 5th.

$2 \times 2 = 4$ square of second power.

$2 \times 2 \times 2 = 8$ cube or third power.

$2 \times 2 \times 2 \times 2 = 16$ biquadrate or 4th power.

$2 \times 2 \times 2 \times 2 \times 2 = 32$; 5th power.

1. A floor is 12 feet square, how many feet of boards does it contain? Ans. 144 ft.

2. In a plantation 300 perches square, how many perches does it contain? Ans. 90000.

TABLE OF POWERS.

Roots	Squares.	Cubes.	4th pow.	5th pow.	6th pow.	7th pow.	8th pow.	9th pr.
1	1	1	1	1	1	1	1	1
2	4	8	16	32	64	128	256	512
3	9	27	81	243	729	2187	6561	19683

SQUARE ROOT.

1. To extract the Square Root prepare the number for extraction by pointing it from units place, into periods of two figures each.

2. Find by involution or for convenience sake in the table, a square nearest to the first, subtract and bring down the next period, which place to the right of the remainder for a dividend.

3. Double the quotient figure for a divisor, and try how often it is contained in all the figures of the dividend except the one, on its right.

4. Place this in the quotient for a second figure of the root, as well as to the right of the divisor.

5. Multiply by this quotient figure as in division, the product subtract as before, and to the difference bring down the third period.

6. Proceed in like manner, still doubling the quotient figures for a new divisor, and bringing down another period each time for a new dividend, until the whole is completed.

EXAMPLE.

1. What is the square root of 10, 34, 26, 56? (3216

9

62)134
124

641)1026
641

6426)38556
38556

2. Extract the square root of 151321? Ans. 389 rt.
3. " " " of 2985984? " 1728 rt.
4. " " " of 23097636? " 4806 rt.

CASE II.

In extracting the root of whole numbers and decimals, one half the number of decimal figures must be pointed out in the quotient.

1. What is the square root of 3271.4007? Ans. 57.19 +
2. " " " " of 4.372594? " 2.091 +

CASE III.

To extract the Square Root of a mixed number.

RULE 1.—Reduce the fractional part of the mixed number to its lowest term, and then the mixed number to an improper fraction.

2. Extract the roots of the numerator and denominator for a *new numerator and denominator.*

3. If the mixed number given be a surd, reduce the fractional part to a decimal, annex it to the whole number, and extract the square root.

1. What is the square root of $42\frac{1}{4}$? Ans. $6\frac{1}{2}$.
2. What is the square root of $51\frac{4}{9}$? " $7\frac{1}{3}$.

CASE IV.

To extract the Square Root of a fraction.

RULE.—Reduce the fraction to its lowest terms, then extract the square root of the numerator for a new numerator, and the denominator for a new denominator.

1. What is the square root of $\frac{4}{9}\frac{9}{16}\frac{4}{4}$? Ans. $\frac{2}{3}$.
2. What is the square root of $\frac{2}{4}\frac{5}{9}\frac{6}{2}\frac{4}{5}$? " $\frac{4}{5}$.

CASE V.

To extract the Square Root of a Surd.

RULE.—Reduce the surd to a decimal, and extract the root thereof.

1. What is the root of $\frac{8}{9}\frac{7}{4}\frac{3}{1}$? Ans. 89802 +
2. What is the root of $\frac{3}{4}\frac{7}{5}\frac{2}{6}$? " 86602 +

CASE VI.

To find a mean proportional.

RULE.—Multiply the two numbers together, and extract the square root of the product for a mean proportional.

1. Suppose A. in a school room sits 4 feet from a hot stove, and B. 9 feet from the same, how much warmer is A. than B? Ans. 6 times.

2. Two ships sail from the same port, one goes due North 128 miles, the other due East 72, how far are the ships asunder? Ans. 146.86 miles.

CASE VII.

The base and perpendicular given to find the hypothenuse.

1. The top of a castle is 45 yards high, and is surrounded with a ditch 60 yards broad, what length must a ladder be to reach from the outside of the ditch to the top of the castle? Ans. 75 yds.

2. The wall of a fort is 25 feet high, which is surrounded by a moat 30 feet in breadth, I want to know the length of a ladder that will reach from the outside of the moat to the top of the wall? Ans. 39.05 ft.

CASE VIII.

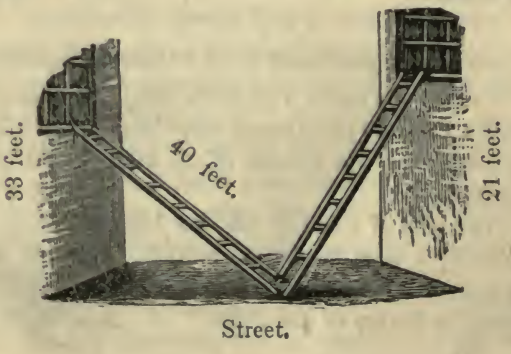
To find the distance that any object may be seen at sea, elevated at any height above the level of the water.

RULE.—Add to the earth's semi-diameter in feet, the height of the object; square the sum, next square the number of miles in the earth's semi-diameter, take the difference of those squares, then by (Euclid 47, Lib. 1,) the square root of the difference of those squares will be the distance required in feet.

1. There is a point of the Andes in South America which is 4 miles above the level of the sea, to what distance could a person see from the top of such an elevated point, provided the atmosphere was perfectly clear, and not assisted by refraction.

Illustration.—If we put the earth's semi-diameter at 4000 miles, then $\sqrt{4004 \times 4004 - 4000 \times 4000} = 178.93 +$ miles, the distance required, which is about twice as far as a person could see elevated from a point one mile above the level of the sea.

2. Suppose a ladder 40 feet long be so planted as to reach a window 33 feet from the ground on one side of the street, and without moving it at the foot, will reach a window on the other side 21 feet high, what is the breadth of the street? Ans. 56, 64 + feet.



CUBE ROOT.

(Sign $\sqrt[3]{\ }$)

Roots.	1	2	3	4	5	6	7	8	9
Cubes.	1	8	27	64	125	216	343	512	729

RULE 1.—Point every third figure beginning at the units place, then find the nearest cube to the first point, and subtract it therefrom; put the root in the quotient, bring down the figure in the next point to the remainder for a dividend.

2. Square the quotient and multiply it by 3, for a divisor; find how often the divisor is contained in the dividend, rejecting units and tens, and place the number of times in the quotient.

3. Square the last figure placed in the quotient, and place the result to the right hand of the divisor, for a defective divisor.

4. Then multiply the last figure placed in the quotient by the other figures, and that product by 30; add the last product to the (defective divisor,) placing units under units and tens under tens, for a complete divisor.

Illustration.—Extract the cube root of

4	$\sqrt[3]{99252847}$(436 root		4	46
4	64		6	3
16 square of 4	35252		24	138
3	33333		30	30
4836 divisor.	1916847		720	4140
720	1916847			
5556				

46	276	2116	634809	638949
46	184	3	4140	

1. What is the cube root of 389017? Ans. 73.
2. What is the cube root of 5735339? " 179.

3. What is the cube root of 32461759? Ans. 319.
4. What is the cube root of 84604519? " 439.
5. What is the cube root of 259694072? " 638.

CASE III.

To extract the Cube Root of a fraction.

RULE.—Reduce the fraction to the lowest denomination. Then extract the Cube Root of the numerator for a new numerator, and also of the denominator for a new denominator.

1. What is the cube root of $\frac{250}{686}$? Ans. $\frac{5}{7}$.
2. What is the cube root of $\frac{324}{1500}$? Ans. $\frac{3}{5}$.
3. What is the cube root of $\frac{1520}{5130}$? Ans. $\frac{2}{3}$.

CASE IV.

To extract the Cube Root of a mixed number.

RULE.—Reduce the fractional part to its lowest terms, and then the mixed number to an improper fraction, extract the roots of the numerator and denominator for a new numerator and denominator, but if the mixed number given be a surd, reduce the fractional part to a decimal, annex it to the whole number, and extract the root therefrom.

1. What is the cube root of $12\frac{19}{27}$? Ans. $2\frac{1}{3}$.
2. What is the cube root of $31\frac{15}{343}$? Ans. $3\frac{1}{7}$.
3. What is the cube root of $405\frac{23}{125}$? Ans. $7\frac{2}{5}$.

APPLICATION.

1. If a cubical piece of timber be 47 inches long, 47 inches broad and 47 inches deep, how many cubical inches does it contain? Ans. 103823.

2. There is a cellar dug, 12 feet in every way, length, breadth and depth, how many solid feet of earth are taken out of it? Ans. 1728.

3. The solid content of a cube is 389017 feet, what is the superficial content of one of its sides? Ans. 5329.

Well known principles assumed.

Circles are to one another as the squares of their diameters. Spheres are to each other as the cubes of their diameters. Cubes and all similar solid bodies as the cubes of their diameters or homologous sides. Whatever constitutes length, breadth and thickness or depth is a solid.

EXAMPLES.

1. If a ball 3 inches in diameter weigh 4 lbs., what will be the weight of a ball that is 6 inches in diameter? Ans. 32 lbs.

2. The solid content of a cellar, which is alike in length, breadth and depth, is 100¾ cubic yards, required the length of its side. Ans. 13.95 feet +

CASE V.

The side of a cube being given to find the side of a cube which shall be double, treble, &c. in quantity to the given cube.

RULE.—Cube the given number and multiply it by 2, 3, &c. the cube root of the product, is the side sought.

1. There is a cubical vessel whose side is 12 inches, it is required to find the side of another vessel that is to contain 3 times as much? Ans. 17.306 inches.

2. If a ship of 400 tons burden be 80 feet long in the keel; what is the burden of another ship the keel of which is 100 feet long? Ans. 781 tons, 5 cwt.

3. The dimensions of a ship are, viz: keel 125 feet long, beam 25 feet, depth of hold 15, what dimensions should a ship of similar form have, to carry 3 times the burden? Ans. length of keel 180,28 feet, breadth of beam 36.05, depth of hold 21.63.

4. Find the dimensions of a similar ship that shall contain, or carry just half the burden of that whose dimensions are given. Ans. length of keel 99.21 feet, beam 19.81, hold 11.09.

5. Suppose a cannon ball of 4 inches diameter, weighs 18 lb.; what is the diameter of another that weighs 42 lbs?

Ans. 5.30 + inches.

6. Suppose a mortar shell of 8 inches diameter, weighs 50 lb; what is the diameter of a shell that weighs 100 lbs?

Ans. 10.08 + inches.

CASE VI.

To find mean proportionals between two given numbers.

RULE.—Multiply the square of the lesser extreme by the greater, the cube root of the product will be the lesser mean. Again, multiply the square of the greater extreme by the lesser extreme, the cube root of the product will be the greater mean.

Example.—Required to find 2 mean proportionals between 4 and 256. Illustration, $\overline{4 \times 4} \times 256 = 4096$ $\sqrt[3]{4096} = 16$. Again, $\overline{256 \times 256} \times 4 = 262144$, the $\sqrt[3]{262144} = 64$, hence 16 and 64 are the mean proportionals required.

SINGLE POSITION.

This rule is called Position because by using supposed numbers according to the conditions of the question, the answer is obtained.

RULE.—As the sum of the errors is to the given sum, so is the supposed number to the true one required.

PROOF.—Add the several parts of the result together, and if it agrees with the given sum it is right.

1. A person, after spending $\frac{1}{3}$ and $\frac{1}{4}$ of his money had 60 dollars left; what had he at first? Ans. $144.

Suppose he had 24

$\frac{1}{3}$	8
$\frac{1}{4}$	6

Spent 14 then 24—14 = 10

Say as 10 : 24 : : 60
1 6 6

Ans. $144 Proof 144

$\frac{1}{3}$ 48
$\frac{1}{4}$ 36

84 then 144—84 = 60

2. A teacher being asked how many scholars he had, said, if I had as many, half as many, and one quarter as many more, I should have 264, how many had he?

Ans. 96 scholars.

3. What number is it whose $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ make 235?

Ans. 300.

4. What is the age of a person who says, that if $\frac{2}{3}$ of the years I have lived be multiplied by 7 and $\frac{3}{4}$ of them be added to the product, the sum will be 292?

Ans. 60 years.

5. A's age is double that of B, and B's is triple that of C. and the sum of all their ages is 140, what is each person's age? Ans. A's 84, B's 42, and C's 14 yrs.

6. A certain sum of money is to be divided among 4 persons in such a manner that the first shall have $\frac{1}{3}$ of it, the second $\frac{1}{4}$, the third $\frac{1}{5}$, and the fourth the remainder, which is $28, what is the sum? Ans. $112.

7. What sum at 6 per cent. per annum, will amount to $1032 in 12 years? Ans. $600.

DOUBLE POSITION.*

Double Position teaches to resolve questions, by making two suppositions of false numbers.

Those questions in which the results are not proportional to their positions, belong to this rule.

* This rule is founded on the suppositon, that the first error is to the second, as the difference between the true and first supposed number is to the difference between the true and second supposed number. When this is not the case, the exact answer to the questions cannot be found by this rule.

1. John asked James how much his horse cost, who answered that if he cost him three times as much as he did, and $15 dollars more, he would stand him in $300, what was the price of the horse? Ans. $95.

Suppose	96	Suppose	90
	3		3
	288		270
More	15	More	15
	303		285
Should be	300	Should be	300
Too much	+ 3	Too little	— 15
	1		5

Now the errors are in the same ratio as one to five.

$$\begin{array}{rcrll} 96 & & 90 & 96 \times 5 = & 480 \\ +1 & X & 5 - & 90 \times 1 = & 90 \\ & & & 6 & 6)570(= \$95 \text{ Ans.} \end{array}$$

The following rule will be found very concise when the signs of + or — are alike subtract, when unlike add:

Illustration. When both signs are + or both signs —, *subtract*, but when one sign is + and the other — *add*. If the errors are alike, divide the difference of the products by the difference of the errors; but, if unlike, divide the sum of the products by the sum of the errors.

1. Divide 15 into 2 such parts, so that when the greater is multiplied by 4, and the less by 16, their products will be equal? Ans. 12 and 3.

2. Divide 21 into 2 such parts, so that when the greater is divided by the lesser, and the lesser by the greater, and afterwards the greater quotient multiplied by 5, and the lesser by 125, their products will be equal?

Ans. $17\frac{1}{2}$ and $3\frac{1}{2}$.

3. A lady being asked her age thus replied:

My age is such, if multiplied by three,
Two-sevenths of that product it tripled be;
The square root of *two-ninths* of that is four,
Now tell my age, or never see me more. Ans. 28 years.

4. A laborer was hired 80 days upon this condition, that for every day he was idle, he should pay his employer 50 cents, and for every day he was at work he should receive 75 cents, at the expiration of the time, he received $25; now, how many days did he work, and how many days was he idle?

Ans. he worked 52 days, and was idle 28.

5. Two persons, A. and B. have the same income, A. saves one-fifth of his every year, but B. by spending $150 per annum more than A. at the end of 8 years finds himself $400 in debt, what is their income, and what does each spend per annum?

Ans. Their income is $500 per annum, A. spends $400, and B. $550.

6. A man had two silver cups of unequal weight, having one cover to both, 5 oz.; now if the cover is put on the lesser cup, the whole will be double the weight of the greater cup. Again, if the cover be put on the greater cup, it will be 3 times as heavy as the lesser cup, what is the weight of each cup?

Ans. 3 ozs. lesser—greater 4 ozs.

7. Divide 10 into 2 such parts, so that 9 times the lesser number will be equal to 6 times the greater?

Ans. 6 and 4.

8. The sum of 2 numbers is 50; now, if you divide the greater part by 7, and multiply the lesser by 3, the sum of the quotient and product will be equal to the given number. Required the parts? Ans. 35 and 15.

9. A young gentleman having asked his father how old he was, received the following reply: seven years ago my age was in a three-fold ratio to yours; but, if we should both happen to live seven years hence, my age shall be just double that of yours. Required their several ages? Ans. 49 and 21 years.

ANNUITIES.

Table First, shewing the amount of $1.00 *annuity from One year to Thirty-seven.*

Yrs.	5 *per cent.*	6 *per cent.*	*Yrs.*	5 *per cent.*	6 *per cent.*	*Yrs.*	5 *per cent.*	6 *per cent.*
1	1,00000	1,00000	16	23,65749	25,67252			
2	2,05600	2,060000	17	25,84036	28,21288			
3	3,152500	3,183600	18	28,13238	30,90565	31	70,76079	84,80167
4	4,310125	4,374616	19	30,53900	33,75999			
5	5,52563	5,63709	20	33,06595	36,78559	32	75,29882	90,88977
6	6,80191	6,97531	21	35,71925	39,99272			
7	8,14200	8,39383	22	38,50521	43,39229	33	80,06377	97,34316
8	9,54910	9,89746	23	41,43047	46,99582			
9	11,02656	11,49131	24	44,50199	50,81557	34	85,06696	104,18375
10	12,57789	13,18079	25	47,72709	54,86451			
11	14,20678	14,97164	26	51,11345	59,15638	35	90,22030	111,43478
12	15,91712	16,86994	27	54,66912	63,70576			
13	17,71298	18,88213	28	58,40258	68,52811	36	95,83632	119,12086
14	19,59863	21,01506	29	62,33271	73,63979			
15	21,57856	23,27597	30	66,43884	79,05816	37	101,62813	127,26811

ANNUITIES AT COMPOUND INTEREST.

DEFINITIONS.—*Annuity* is a certain sum of money to be paid at regular periods, either for a limited time or for ever.

Present worth, or value of an annuity is that sum, which being improved at compound interest, will be sufficient to pay the annuity.

The amount, of an annuity is the compound interest of each payment added to their sum. To find the amount of an annuity at compound interest, we adopt the following.

RULE.—Make $1 the first term of a geometrical series, and the amount of $1 at the given rate per cent. the ratio, carry the series to so many terms as the number of years, and find its sum. Multiply the sum thus found by the given annuity and the product will be the amount.

CASE I.

Example.—1. What will an annuity of $60 per annum, payable yearly, amount to in 4 years, at 6 per cent?

$$1\times1.06\times1.06 + 1.06 = 4.37461,$$

the tabular number, answering to 4 years, at 6 per cent.

Table 1, $\overline{1.06^4-1}$

$1.06—1\times60 = \$262.47c.\ 6m.$ Ans.

or $4.374616\times60 = \$262.47c.\ 6m.$ as before.

2. What will an annuity of $30 amount to in 3 years?
Ans. $95.50c. 8m.

3. What will an annuity of $200 amount to in 5 years at 6 per cent? Ans. $1127.41c. 8m.

CASE II.

When the payments are to be made half yearly or quarterly, the amount for the given time, found as before, in table first, multiplied by the tabular number answering

to the given rate per cent. for quarterly or half yearly payments, will be the true amount.

TABLE SECOND.

Rate per cent.	*Half yearly payments.*	*Quarterly payments.*
3	1,007445	1,011181
3½	1,008675	1,013031
4	1,009902	1,014877
4½	1,011126	1,016720
5	1,012348	1,018559
5½	1,013567	1,020395
6	1,014781	1,022257
7	1,017204	1,025880

The construction of this table is from an Algebraic Theorem given by the learned *Mons. De Moivre*, in his Treatise of Annuities on Lives, which in words is thus: For half yearly payments take a unit from the ratio, and from the square root of the ratio, half the quotient of the first remainder, divided by the latter will be the tabular number; for quarterly payments, use the fourth root, as above, and take one-fourth of the quotient.

1. What will an annuity of $200 amount to in 5 years, to be paid in half yearly payments, at 6 per cent. per annum? Ans. $1144 8*c*. 2*m*. +

Agreeably to table first, the tabular number is 5.63709 × 200 = $1127 41*c*. 8*m*. × 1.014781, (tabular number answering to 6 per cent. in table second, for half yearly payments,) = $1144 08.2*m*. + answer.

2. What will an annuity of $500 amount to in 5 years, at 6 per cent.? Ans. $2818.54.6 +.

3. What will an annuity of $1000, payable yearly, amount to in 10 years? Ans. $13180.79.4 +.

4. What will annuity of $30, payable yearly, amount to in 3 years? Ans. $95.50.8 +.

ANNUITIES.

Table Third, shewing the present worth of an Annuity of $1, from One year to Thirty-seven.

Yrs.	5 *per cent.*	6 *per cent.*	*Yrs.*	5 *per cent.*	6 *per cent.*	*Yrs.*	5 *per cent.*	6 *per cent.*
1	0,952381	0,943396	14	9,898641	9,294984			
2	1,859410	1,833393	15	10,379658	9,712249	27	14,643034	13,210534
3	2,723248	2,673012	16	10,83770	10,105895	28	14,898127	13,406164
4	3,545950	3,465106	17	11,274066	10,477260	29	15,141074	13,590721
5	4,329477	4,212364	18	11,689587	10,827603	30	15,372451	13,764831
6	5,075692	4,917324	19	12,085321	11,158116	31	15,592810	13,929086
7	5,786373	5,582381	20	12,462216	11,469921	32	15,802677	14,084043
8	6,463213	6,209794	21	12,821153	11,764077	33	16,002549	14,230230
9	7,107822	6,801692	22	13,163003	12,041582	34	16,192904	14,368141
10	7,721735	7,360087	23	13,488574	12,303379	35	16,374194	14,498246
11	8,306414	7,886875	24	13,798642	12,550358	36	16,546852	14,620987
12	8,863252	8,388844	25	14,093945	12,783356	37	16,711287	14,736780
13	9,393573	8,852683	26	14,375185	13,003166			

EXPLANATION OF THE TABLE.

What is the present worth of $1, to continue for 4 years at 6 per cent. per annum? Ans. 3.465106, agreeing with the tabular number opposite to 4 years at 6 per cent. per annum.

First, find the present worth of $1, by discount for 1 year at 6 per ct. per annum, which is $0.943396

2d year the present worth is	-	0.889996
3d " " " is	-	0.839619
4th " " " is	-	0.792094
		$3.465105

Tabular number for 4 years at 6 per. cent. as in table 3d.

1. What is the present worth of $50 per annum for 6 years at 6 per cent. per annum? Ans. $245.86c. 6m.

CASE III.

Annuities in Reversion.

The annuity, time, and rate given, to find the present worth as in *case* 2. Multiply the number, under the rate and opposite th etime in table 3d, by the annuity, the product will be the present worth for yearly payments. If the payments are to be made half-yearly, or quarterly, the present worth so found for yearly payments, must be multiplied by the proper number in table 2d.

Q. What is meant by annuities in reversion?

A. Sums of money, which are paid yearly for a limited period, but which do not commence till after the expiration of a given period, are called annuities in reversion.

Given the time of reversion, time of continuance and rate per cent. to find the present worth of the reversion.

RULE.—Take two numbers under the given rate in table 3, corresponding to the different periods of time, viz; time of reversion and time of continuance, and take the difference between the tabular numbers, answering to the times as above mentioned, and multiply that difference by the

annuity, for the present worth annually, if the payments be half yearly or quarterly, we must use table 2 as above stated.

1. The reversion of a freehold is $60 per annum for 4 years, to commence 2 years hence, what is the present worth, allowing 6 per cent. for prompt payment.

Illustration.—Time of continuance 6 years.

Tabular number for 6 years at 6 per cent. found in table 3, is - - - - - - 4.917324

Time of reversion; 2 years tabular No. 1.833393

3.083931

3.083931 × 60 = $185.03*c*.5*m*. × Answer.

2. What is the present worth of a reversion of a lease of $100 per annum, to continue 10 years; but is not to commence till the end of 2 years at 6 per cent.

Ans. $655.04¼.

3. What is the present worth of a reversion of a lease for $120 per annum to continue 9 years, but not to commence till the end of 4 years at 5 per cent. to the purchaser? Ans. $701.71*c*. 4*m*.

PERPETUITIES.

Annuities which continue for ever, are called perpetuities.

CASE IV.

Given the Annuity and rate per cent. to find the present worth.

RULE.—Divide the annuity by the ratio less 1, for the present worth.

Note.—Table 2d must be resorted to, as in temporary annuities, when the payments are half-yearly or quarterly.

1. What is the present worth of an annuity of $150 to continue for ever, allowing 5 per cent. to the purchaser?

Operation.—1.05—1 = .05)150.00

$3000 Ans.

2. What is an estate of $260 per annum, to continue for ever, worth in present money, allowing 6 per cent. to the purchaser? Ans. $4333.33.3 +.

3. A property in fee simple rents for $120 per annum, what is the present worth, allowing 5 per cent. to the purchaser? Ans. $2400.

DISCOUNT BY COMPOUND INTEREST.

The ratio in compound interest is the amount of $1 for one year, which is found thus: as 100 : 106 : : 1 = $1.06 is the amount of $1 for one year.

Example 1.—What is the present worth of 600 for 3 years; hence at 6 per cent. compound interest $\overline{1.06^3}$ = 1.191016)600(= $503.77 + Ans.

2. What is the amount of $503.77, in 3 years at 6 per cent.? Ans. $600.

3. What is the present worth of $520, due 5 years hence, at 6 per cent. compound interest. Here $\frac{520}{1.06^5}$ = 1,3312256)520(=390.62 answer.

ALLIGATION,

From the Latin (*ad.* to, and *ligo* to bind,) it being necessary in sundry cases to link or bind the quantities.

We shall not omit the rule of Alligation, the object of which is to find the value of several things of the same kind of different values. The following examples will sufficiently demonstrate it.

CASE I.

When the quantities and rates of the simples are given to find the rate of a mixture compounded of these simples.

RULE.—Find the value of each quantity, according to their respective costs, then divide the sum of the pro-

ducts by the aggregate of the quantities, and the quotient will be the average value of each quantity.

1. A wine merchant bought several kinds of wine, viz:
130 bottles which cost him 10 cents each.
75 " at 15 cents each.
231 " at 12 " "
27 " at 20 " "

Now, 130 at 10 cts. = 1300
75 " 15 " = 1125
231 " 12 " = 2772
27 " 20 " = 540
463 bottles cost 5737 cts.

463)5737(12 39
463
1107
926
1810
1389
4210
3867

2. A grocer has 4 lbs. of tea at 90 cents per lb., 8 lbs. at 75 cents, and 6 lbs. at 110 cents. to be mixed together, what will a pound of this mixture be worth? Ans. 90c.

3. A grocer has 2 cwt. of coffee at $25 per cwt.; 4 cwt. at $20.50 per cwt. and 7 cwt. at $18.62½ per cwt. which he will mix together, what will 1 cwt. of this mixture be worth? Ans. $20.18¾.

CASE II.

When the prices of all the simples, the quantity of one of them, and the mean price of the whole mixture are given to find the quantities of all the rest.

RULE 1.—Place the mean rate and the several prices, link them and take their differences, as in the preceding case.

2. As the difference of the same name with the quantity given is to the differences respectively, so is the given quantity to the several required quantities.

1. What quantity of coffee at 20 cents, and at 16 cts. per lb. must be mixed with 35 lb. at 14 cents to make a mixture worth 18 cents per lb.?

Mean rate 18 { 14, 16, 20 } { 2, 2, 4 + 2 = 6 }

Then, as 2 : 35 : : 2 = 35 at 16.
2 : 35 : : 6 = 105 at 20.

2. How much tea at 86 cents, at 94 cents, and at 105 cents per lb. ought to be mixed with 6 lbs. at 75 cts. per lb. for a mixture, to sell at 92 cts. per lb.?

Ans. 18 lbs. at $1.05, 51 lbs. at 94c., 39 lbs. at 86c.

CASE III.

When the prices of the several simples, the quantity to be compounded, and the mean price are given to find the quantity of each simple.

RULE 1.—Link the several prices and take their differences as before. 2d. As the sum of the differences is to the difference opposite each price, so is the quantity to be compounded to the quantity required.

1. A grocer has three sorts of sugar, viz: 10, 11, and 8 cents per lb. how much of each sort must he take?

Mean rate 9 { 8, 10, 11 } { 1 + 2 = 3, = 1 = 1, = 1 = 1 }

Sum of differences, - 5

As 5 : 3 : : 40 = 24 at 8 cents.
5 : 1 : : 40 = 8 at 10 "
5 : 1 : : 40 = 8 at 11 "

2. A vintner has wine at 130 cts. at 160 cts. and at 180 cts. per gallon, and he would have 32 gallons worth 145 cents per gallon, I demand how much of each sort he must have? Ans. 20 gals. at $1.30, 6 gals. at $1.60 and 6 gals. at $1.80.

ARITHMETICAL PROGRESSION.

When a series of numbers or quantities increase or decrease by a constant difference, it is called Arithmetical progression; as, 1, 2, 3, 4, 5 6; 1, 3, 5, 7, 9, 11; 6, 5, 4, 3, 2, 1; 11, 9, 7, 5, 3, 1. There are five things to be particularly attended to in Arithmetical Progression; the first term, the last term, the number of terms, the common difference, and the sum of all the terms.

CASE I.

The first term, common difference, and number of terms being given to find the last term, and sum of all the terms.

RULE 1.—Multiply the number of terms, less one, by the common difference, and to that product, add the first term, the sum is the last term.

2. Add the first and last terms together, and multiply the sum by the number of terms, and half the product will be the sum of all the terms.

1. A person sold 40 yards of muslin at 2 cents for the first yard, 4 cents for the second, increasing 2 cents every yard, what did they amount to? Ans. $16.40.

OPERATION.

Nos. of terms $40 - 1 = 39 \times 2 = 78 + 2 = 80$ last term.

1st term or extreme 2

Last do. or second extreme 80

$82 \times 40 = \frac{3280}{2} = \$16.40$

2. A butcher bought 75 sheep, and gave 6 cents for the first, 8 for the second, &c., what did he give for the last, and what did the whole number cost him?

Ans. For the last $1.54, the whole $60.

3. A travels uniformly at the rate of 6 miles an hour, and sets off upon his journey 3 hours and 20 minutes before B; B follows him at the rate of 5 miles the first hour, 6 the second, 7 the third, and so on. In how many hours will B overtake A? Ans. 8 hours.

CASE II.

When the first and last terms (or two extremes) are given to find the common difference.

RULE.—Divide the difference of the extremes by the number of terms less 1; the quotient will be the common difference.

1. If the ages of 12 persons are equally different, the youngest is 18 years and the eldest 40, what is the common difference of their age? Ans. 2 common difference.

Illustration of the above question.

40
18
—
12—1 = 11)22(2 common difference.
22
—

2. The extremes are 3 and 45, and the number of terms is 22, what is the common difference? Ans. 2.

3. A man received "charity" from 10 different persons, the first 4 cents, the last 49 cents, what was the common difference, and what did the man receive?

Ans. he received $2.65; com. dif. 5 cts.

4. The extremes are 3 and 39, and the sum of the series 399; what is the common difference? Ans. 2.

GEOMETRICAL PROGRESSION.

Geometrical Progression is the increase of a series of numbers by a common multiplier, or decrease by a common divisor; as 2, 4, 8, 16, 32; 32, 16, 8, 4, 2; the ratio is the number by which the series increases or decreases.

CASE I.

To find the last term and sum of the series.

RULE.—Raise the ratio to the power whose index is 1 less than the number of terms given.

2. Multiply the product by the first term, and the result will be the last term.

3. Multiply the last term by the ratio; from the product subtract the first term, and divide the remainder by the ratio less 1, for the sum of the series.

1. If I buy 16 cords of wood, and agree to pay 2 cents for the first, 4 for the second, 8 for the third, &c., doubling the price to the last, what will it cost me?

1*st*. 2*nd*. 3*rd*. 4*th*.
power 1, 2, 3, 4, ratio 2, 4, 8, 16

fourth power	16	
	96	
	16	
	256	8th power.
fourth power	16	
	1536	
	256	
	4096	12th power.
third power	8	
	32768	15th power.
first term	2	
	65526	last term.
ratio	2	
	131072	
	2	first term.
Ratio 2—1 = 1)	131070	
	$1310.70	Answer.

2. A person at the birth of his son, deposited in bank 1 cent, towards his fortune, promising to double it at the return of every birthday, until he was 21 years of age, what was his portion? Ans. $20,971.51.

3. A gentleman consented to have his daughter married on New Year's day, and agreed to give her one dollar towards her portion, promising to double it, on the first day of every month for one year, what was her portion?

Ans. $4095.

4. A thresher wrought 20 days and received for the first day's labour 4 grains of wheat; for the second 12; for the third 36, &c, how much did his wages amount to, allowing 7680 grains to make a pint, and the whole to be disposed of at $1 per bushel? Ans. $14187.

5. A sum of money is to be divided among 10 persons, the first is to have $10, the second $30 and so on in three fold proportion; what will the last have? Ans. $196830.

DUODECIMALS.

This rule is of great use to carpenters, joiners, &c. The name is derived from the latin words *duo*, 12 and *decem*, 10, and as the ratio is 12, it may with propriety be termed *Duodecimals*. As the French and other European nations divide their inch into 12 equal lines, so our American artificers, suppose the inch to be divided as follows:

DENOMINATIONS OF DUODECIMALS.

12 fourths make	1 third,	fourths marked thus		''''
12 thirds make	1 second,	thirds	"	'''
12 seconds "	1 inch,	seconds	"	''
12 inches "	1 foot,	inches	"	I
		Feet	"	F

EXAMPLES IN ADDITION.

The ratio being 12 the rule is evident.

F.	I.	''	'''
Add 642.	11.	8.	8
436.	10.	6.	4
Add 6468.	10.	5	
4864.	9.	3	
6968.	8.	7	

F.	I.	''
Add 6464.	10.	9
4243.	9.	4
Add 9346.	7.	3
4842.	9.	4
3464.	11.	6

MULTIPLICATION.—*Observe the following rules.*

Feet multiplied by feet produce feet.
Feet multiplied by inches produce inches.
Feet multiplied by seconds produce seconds.
Inches multiplied by inches produce seconds.
Inches multiplied by seconds produce thirds.
Seconds multiplied by seconds produce fourths.

1. Multiply 5 feet 6 inches by 2 feet 4 inches.

F	*I.*
5.	6
2.	4
11.	0
1.	10.0
12.	10.0

2. Multiply 8 feet 6 inches by 14 feet 9 inches.

Ans. 125 ft. 4 in. 6 s.

CASE II.

RULE.—Multiply by the component parts, as in compound multiplication, and take parts for the inches as in practice.

1. Multiply 208 feet 8 inches 4 seconds, by 24 feet 3 inches 9 seconds.

	F.	*I.*	″
Operation.	208.	8.	4
			6 × 4
	1252.	2.	0
			4
	5008.	8.	0
3 in. is ¼	52.	2.	1
9 ″ is ¼	13.	0.	6.3
	5073,	10.	7.3 Answer.

2. Multiply 4 feet 7 inches by 6 feet 4 inches.

Ans. 29 ft. 0. in 4 s.

3. How many square feet are in a floor 48 ft. 6 in. long, by 24 ft. 3 in. broad? Ans. 1176 ft. 1 in. 6 s.

4. How many cubic feet of stone work are contained in 9 walls, each 30 ft. 6 in. long, 9 ft. 8 in. high and 2 ft. thick? Ans. 5307 ft.

5. How many cubic feet in a cellar 30 ft. 3 in. long, 27 ft. 6 in. broad, and 8 ft. deep? Ans. 6655 ft.

6. What is the content of a marble slab 6 ft. 7 in. long, 2 ft. broad, and 1 ft. 9 in. thick? Ans. 23 ft. 0 in. 6 s.

7. What is the superficial content of a stone 4 ft. 9 in. long, and 3 ft. 9½ in. broad? Ans. 18 ft. 0 in. 1 s. 6 t.

MENSURATION.

IMPORTANT TO SHIP BUILDERS.

A concise Rule to find the length of Masts.

RULE 1.—Multiply the length of the keel by 2, and divide the product by 3, and then to the quotient add the breadth of the beam, and the sum will be the length of the main mast.

1. Suppose a ship to be 84 feet by the keel, and 31 ft. by the beam, what is the length of her mast? Ans. 87 ft.

2. Suppose a ship to be 108 feet by the keel, and 40 feet by the beam, what is the length of her main mast? Ans. 112 feet.

ANOTHER METHOD.

When the length and thickness of Masts is required in yards.

RULE.—Add the breadth of the beam and the depth of the hold in feet together, and divide the sum by 1.5, and the quotient will be the length of the main mast in yards.

1. Admit a ship whose keel in length is 73 feet, and the breadth of the beam 28.5 feet, and the depth of the hold 12 feet, what is the length of her main mast? Ans. 81 feet.

	Feet.
Breadth of the beam,	28.5
Depth of the hold,	12.0

1.5)40.5(27 yards or 81 feet.

Now to find the thickness, it is customary to allow $\frac{1}{3}$ of the length in feet for the thickness in inches; in that case, a main mast 81 feet long, must be 27 inches thick.

2. If a ship of 100 tons be 44 feet long at the keel, of what length must the keel of a ship be that carries 220 tons? *tons. tons.*

Say as 100 : 220 : : 44^3 = (85184) 187404.80, whose cube root is 57.226, the length of the keel sought.

To find Ship's Tonnage by Carpenter's Measure.

RULE.—For *single-decked vessels*, multiply the length and breadth at the main beam, and depth of the hold together, and divide the product by 95.

For *double-decked vessels*, take half the breadth of the main beam, and work as above directed.

1. What is the tonnage of a single decked vessel, whose length is 60 feet, breadth 20, and depth 8 feet?

Ans. 101 $\frac{1}{19}$ tons.

Operation.—$\frac{60 \times 20 \times 8}{95} = 101\ \frac{1}{19}$ tons.

2. What is the tonnage of a double-decked vessel, whose length is 65 feet, breadth 21 feet 6 inches, and depth 10 feet 9 inches? Ans. 158$\frac{1}{2}\frac{1}{3}$ tons.

TO FIND GOVERNMENT TONNAGE.

GOVERNMENT RULE.—"If the vessel be double-decked, take the length thereof from the fore part of the main stem to the after part of the stern-post above the upper deck; the breadth thereof at the broadest part above the main wales, half of which breadth shall be accounted the depth of such vessel, and then deduct from the length $\frac{4}{5}$ of the breadth; multiply the remainder by the breadth, and the product by the depth, and divide this last product by 95, the quotient whereof shall be deemed the true contents or tonnage of such ship or vessel; and if such ship or vessel be single-decked, take the length and

breadth, as above directed, deduct from the said length $\frac{3}{5}$ of the breadth, and take the depth from the under side of the deck-plank to the ceiling in the hold, and the multiply and divide as aforesaid, and the quotient shall be deemed the tonnage."

1. What is the government tonnage of a single-decked vessel, whose length is 69 feet 6 inches, breadth 22 feet 6 inches, and depth 8 feet 6 inches?

	F. I.		F. I.
Illustration of the rule	69.6	breadth	22.6
Deduct	13.6		3
	56.0		5)67.6
Breadth	22.6		13.6
	112.0		
	112		
	1232.0		
6 in. is $\frac{1}{2}$	28.0		

$$\text{feet } 1260 \times \overset{f.\,.i}{8.6} = \overset{feet.}{\tfrac{10710}{95}} = \overset{tons.}{112\tfrac{14}{19}}$$

2. Required the tonnage of a single decked vessel, by carpenter's measure, whose length is 70 feet 6 inches, breadth 24 feet 8 inches, and depth 9 feet 10 inches?

Ans. 180 tons.

3. What is the Government tonnage of a double-decked vessel of the following dimensions: length 82 feet 3 inches, breadth 24 feet 3 inches, and depth 12 feet $1\frac{1}{2}$ inches? Ans. $209\frac{63}{95}$ tons.

BOARD OR LUMBER MEASURE.

Rule.—Multiply the length in feet by its breadth in inches, and divide by 12 for the content.

1. What is the content of a board 24 feet long and 8 inches wide? Ans. 16 ft.

2. What is the content of a board 30 feet by 16 inches? Ans. 40.

3. What is the content of a board 14 feet by 15 inches? Ans. 17½ ft.

4. What is the content of a board 18 feet by 15 inches? Ans. 22½ ft.

TO MEASURE SCANTLING OR JOIST.

RULE.—Multiply the depth and width taken in inches by the length in feet, divide the product by 12, and the quotient is the content in feet.

1. How many feet are there in 3 joist, each of which are 15 feet long, 5 inches wide and 3 inches thick? Ans. 56¼ feet.

2. How many feet in 20 joist 10 feet long 6 inches wide, and 2 inches thick? Ans. 200 feet.

CASE II.

When a Board is wider at one end than the other.

GENERAL RULE.—Take the breadth in the middle, or add the measure of both ends together, and take the sum for a mean breadth which multiply by the length for the content.

1. Suppose a board be 10 feet long and 10 inches wide at one end, and 34 inches wide at the other end, what is its superficial content? Ans. 18½ feet.

Illustration,

	34
	10
½ sum,	44
	22 medium or mean breadth.
	10 feet long
12)	220
	18½ feet answer;

or, if the length be in feet and inches, reduce the length to inches, which being multiplied by the mean breadth in inches and divided by 144, we get the content in feet.

Q. Why do we divide by 144?

A. Because, when we multiply inches by inches, the product is square inches; therefore, we divide by 144, 144 square inches being = to 1 square foot.

PAPERING ROOMS.

There is a room papered, the compass of which is 47 feet 3 inches, and the height 7 feet 6 inches, what is the content in square yards? Ans. 39½ yds.

CARPENTER'S WORK.

Roofing, flooring, partitioning, and the principal carpentry in modern buildings, are measured by the square of 10 feet, that is 100 feet.

RULE FOR ROOFING.—Multiply the depth and half depth by the front, or, the front and half front by the depth, and you get the content. The dimensions are taken in feet and inches.

1. If a floor be 49 feet 6 inches long, and 26 feet 6 inches broad, how many square feet?

Operation.

49.5
26.5

2475
2970
990

1311(75 Ans. 13,11 ft. 9 in. or 13s. 11f. 9i.
12

9)00

BRICKLAYER'S WORK.

Bricklayers are generally paid by the day or perch.

1. Suppose a garden wall to be 254 feet round and 12 feet 7 inches high and 3 bricks thick, how many square rods does it contain? Ans. 23¼ sq. rods.

$$\underset{F.}{254}+\underset{F.\ I.}{12.7} = 3196.2\times 2 = \frac{6392.4}{272} = 23\tfrac{1}{2} \text{ sq. rods.}$$

Note.—As the standard thickness is 1½ brick thick, then 16½ feet long, 1 foot thick and 1½ feet high = 24.75 feet, or 1 perch, hence we multiply the length, breadth and thickness of the wall together, and divide by 24.75 for the number of perches required.

DIGGING.

Cellars, vaults, clay for brick, canals, &c., are measured by the solid yard of 27 feet.

RULE.—Multiply the length, width and depth, together and divide the product by 27, for the number of cubic yards.

How many yards of digging in a cellar 25 feet long, 20 feet wide and 10 feet 6 inches deep? Ans. 194⅓ cub. yds.

Note.—A solid yard of clay will make 7 or 800 brick, and 3½ bushels of lime and half a load of sand will be sufficient to lay 1000 brick.

To find how many thousand brick will be required for building a house of any given dimensions.

Suppose a house of the following dimensions, viz: 84 feet long, 40 feet wide, 20 feet high and the walls to be 1 foot thick? Ans. 105,408 bricks.

RULE.—Deduct the thickness of the wall, from the length of each side, because the inner sides are 1 foot less in height than the outer sides. This rule is unquestionably correct.

OPERATION.

84—1 = 83 feet and 40—1 = 39

Now 83 × 2 = 166 sum of 2 sides in length,
and 39 × 2 = 78 sum of 2 do. in breadth,

244
compass 20 feet in height.

4880 × 1728 = 8432640 cub. in.

Now allowing a brick to be 8 inches long, 4 inches

wide, and 2¼ inches thick, there would be 80 cubic inches in a brick, hence 843264(0 = 105,408 brick, Ans.

8(0

2. How many thousand brick 8 inches long, 4 inches wide, and 2¼ inches thick, will build a wall in front of a church which is to be in compass 240 feet long, 6 feet high and 1 foot 6 inches wide? Ans. 51.840 bricks.

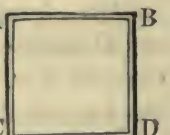
3. How many shingles will it take to cover the roof of a barn 40 feet long, allowing the length of the rafters to be 16 feet 6 inches, and 6 shingles to cover 1 square foot; what will they cost at $1.25 per 1000?

Ans. 7,920 shingles; cost $9.90.

MENSURATION OF SUPERFICES AND SOLIDS.

PROBLEM 1.

To find the area of a square.



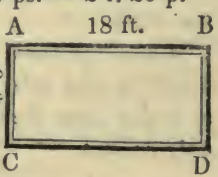
RULE.—Square the side: and the product or rectangle will be the superficial content.

1. A lot of ground is 10 perches square, what is the area?

PROBLEM 2.

Ans. 100 ps. = 2 r. 20 p.

Note.—A square is a parallelogram, but a parallelogram is not a square, because it is an oblong whose length and breadth are unequal.



2. What is the content of a board 15 feet long and 2 feet wide? Ans. 30 feet.

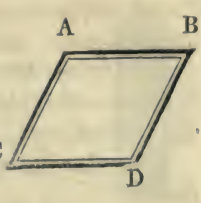
3. What is the difference between a floor 40 feet square and 2 others, each 20 feet square? Ans. 800 feet.

4. There is a square of 3600 yards area; what is the side of a square, and the breadth of a walk along each side of the square at each end, which may take up just one half the square?

Ans. 42.42 + yds. side of the sq. 8.78 + yds. breadth of walk.

PROBLEM 3.

To find the area of a rhombus.

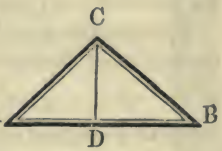


RULE.—Multiply the length of the base by the perpendicular height.

5. The base of a rhombus is 14 feet C and its height 6 feet, required the area? Ans. 84 feet.

PROBLEM 4.

To find the area of a triangle.



RULE.—Multiply half the base by the perpendicular height, or if the perpendicular is not given, add the three sides together, take half that sum, subtract each side severally; from the half sum multiply the half sum and the three differences together, and the square root of the product will be the area.

1. Required the area of a right angled triangle whose base is 40 and perpendicular 30 perches? Ans. 600.

2. Required the area of a triangle whose sides are 10, 12 and 18 perches respectively? Ans. 56.57 perches.

PROBLEM 5.

CASE I.

By having the diameter of a circle to find the area.

RULE.—Square the diameter, and multiply the product by .7854 for the area.

CASE II.

By having the circumference of a circle to find the area.

RULE.—Square the circumference and multiply that square by .07958.

1. The diameter of a circle is 24, required the area? Ans. 452.4904.

2. The circumference of a circle is 80, required the area? Ans. 509.312.

CASE III.

By having the diameter to find the circumference.

RULE.—Multiply the diameter by 3.1416, and you get the circumference.

If the diameter of a circle be 24, what is the circumference? Ans. 75.3984.

CASE IV.

By having the circumference of a cube to find the diameter.

RULE.—Multiply the circumference by .31831 and the product is the diameter.

PROBLEM 6.

I have a circular field 50 rods in diameter, what is the side of a square field, that shall contain the same area?

Ans. 44.31.

SOLIDS

Are figures having length breadth and thickness.

PROBLEM 7.

To find the content of a cube or parallelopipedon, whose side is 18 *inches.*

RULE.—Multiply the length, height, and breadth continually together, and the product is the content.

1. How many cubic feet in a cube whose side is 18 inches? Ans. $3\frac{3}{4}$ feet.

2. What is the content of a parallelopipedon whose length is 6 feet, height 2 feet, breadth $1\frac{1}{2}$ feet? Ans 18ft.

3. A cellar is 50 feet long, 38 feet wide and 12 feet deep, how many cubic yards of earth has been taken out in digging, and what was the expense of digging it at 10 cts. per cubic yard?

Ans. 844.44 + cubic yds.; expense $84.45 nearly.

PROBLEM 8.

To find the solidity of a Prism.

RULE.—Multiply the area of a base or end by the height.

4. Required the solidity of a triangular prism whose length is 10 feet, and the three sides of its triangular base are 5, 4 and 3 ft. Ans. 60ft.

PROBLEM 9.

To find the solidity of a Cylinder.

RULE.—Multiply the area of the base by the length.

5. The diameter of the base of a cylinder is 10 inches and its length 24 feet, required the solidity?

Ans. 13.09 feet.

PROBLEM 10.

To find the solidity of a cone or pyramid.

RULE.—Multiply the area of the base by $\frac{1}{3}$ of its height.

6. What is the solid content of a cone whose heighth is $12\frac{1}{2}$ feet, and the diameter of the base $2\frac{1}{2}$ feet?

Ans. 20.45 + feet.

PROBLEM 11.

To find the superfices of a Cone.

RULE.—Multiply the circumference of the base by half its slant height.

7. What is the convex surface of a cone, whose slant height is is 20 feet, and the circumference of its base 9 feet?

Ans. 90 feet.

PROBLEM 12.

To find the solidity of the frustum of a cone or pyramid.

RULE.—Multiply the diameters of the two bases together, and to the product add $\frac{1}{3}$ of the square of the difference of the diameters; then multiply this sum by .7854 and

the product will be the mean area between the two bases; lastly, multiply the mean area by the length of the frustum, and the product will be the solid content.

Frustum of a cone.

8. What is the content of a stick of timber whose length is 40 feet, the diameter of the larger end 24 inches, and the smaller end 12 inches? Ans. 73½ ft. nearly.

PROBLEM 13.

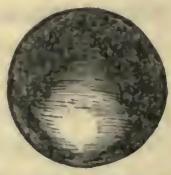
To find the solidity of a sphere or globe.

RULE.—Multiply the cube of the diameter by .5236.

9. What is the solidity of a sphere or globe, whose axis or diameter is 12 inches? Ans. 904.78 + inches.

PROBLEM 14.

To find the convex surface of a sphere or globe.



RULE.—Multiply the diameter by the circumference.

10. Required the superficial content of a globe whose diameter is 24 inches? Ans. 1809.55 + inches.

To find how large a cube may be cut from any given sphere, or be inscribed in it.

RULE.—Square the diameter of the sphere, divide that product by 3, and extract the square root of the quotient for the answer.

1. How large a cube may be inscribed in a sphere 40 inches in diameter? Ans. 23.09 + inches.

GUAGING.

The business of cask guaging is commonly performed by two instruments, namely, the guaging or sliding rule, and the guaging or diagonal rod.

Cask of the common form.

To find the solid content of Casks.

TABLE.

282 Cubic inches for Ale gals.	231 Cubic inches for Wine.	268.8 for Malt gallons.	2150.42 Cubic inches in a bush.	227 Cubic inches for mash tun gal.

RULE.—Find the mean diameter of the cask, by taking the mean of the bilge and head diameter, then multiply the mean diameter in inches by the length of the cask in inches. And, again, multiply this product by the mean diameter; deduct one-fifth of the sum so found for the roundness of the cask, and reduce the remainder to feet and inches, by the rule for measurement, which is correct if the content be required in solid feet, or, divide by 232 for ale gallons.

Example 1.—If a cask be in length 3 feet 9 inches, its head diameter 2 feet 6 inches, and its bung diameter 2 feet 10 inches, what are its solid contents in ale gallons?

Length 3 ft. 9 in.		Multiply 45	length.
12		32	
45		90	
Head diameter, 2 ft. 6 in.	Bung diameter. 2 ft. 10 in.	135	
12	12	1440	
30 Bung diam.	34 inches	32	Again to be multiplied.
Head "	30	2880	
		4320	
	½)64	46080	
Mean diam. 32 in.	Deduct ⅕	9216	
		282)36864(130	gals. qts. 3

2. Each side of the square base of a vessel is 40 inches, and its depth 10 inches. Required its contents in ale gallons? Ans. 56.7 gallons.

8. The diameter of a cylindrical vessel is 32 inches, and its internal depth 45.5 inches. Required its content in ale gallons? Ans. 129.78 gals.

4. How many bushels of grain will a box contain that is 15 feet long, 5 feet wide, and 7 feet high?

Ans. 421.8 bushels.

ABSTRACT OF MECHANICS.

OF MATTER.

MATTER, is possessed of the following properties, viz: *solidity, extension, divisibility, mobility, inertia, attraction* and *repulsion.*

1. SOLIDITY is that property by which two bodies cannot occupy the same place at the same time. It is sometimes called the impenetrability of matter.

2. EXTENSION, like the solidity of matter, is proved by the impossibility of two bodies co-existing in the same place.

3. DIVISIBILITY, is that property by which bodies are capable of being divided into parts removable from each other.

4. MOBILITY expresses the capacity of matter to be moved from one position or part of space to another.

5. INERTIA, designates the passiveness of matter, which, if at rest, will forever remain in that state until compelled by some cause to move; and, on the contrary, if in motion, that motion will not cease, or abate, or change its direction unless the body be resisted.

1. SPACE is either *absolute* or *relative.*

2. ABSOLUTE SPACE, is merely extension, immoveable, illimitable and without parts, yet it is usually spoken of as if it had parts. Hence the expression:

RELATIVE SPACE, signifies that part of absolute space occupied by any body, as compared with any part occupied by another body.

ATTRACTION.

4. ATTRACTION denotes the property which bodies have to approach to each other.

There are five kinds of Attraction: 1, the attraction of *cohesion;* 2, of *gravitation;* 3, of *electricity;* 4, of *magnetism;* 5, of *chemical attraction.*

Cohesion is exerted only at very small distances, its strength varies in different kinds of matter, and is supposed to be the cause of the relative degrees of hardness of different bodies.

Capilliary Attraction is only a particular modification, or branch of the attraction of cohesion.

GRAVITATION decreases from the surface of the earth *upwards*, as the square of the distance increases; but from the surface of the earth *downwards*, it decreases only in a direct ratio to the distance from the centre.

REPULSION.

1. REPULSION is that property in bodies, whereby, if they are placed just beyond the sphere of each other's attraction of cohesion, they mutually fly from each other.

2. OIL refuses to mix with water, from the repulsion between the particles of the two substances; and from the same cause, a needle gently laid upon water will swim.

MOTION.

Absolute Motion, is the actual motion that bodies have, considered independently of each other, and only with regard to the parts of space.

Relative motion, is the degree and direction of the motion of one body, when compared with that of another.

Accelerated motion, is understood, when its velocity continually increases.

Retarded motion, when the velocity continually decreases, and the motion is said to be uniformly retarded. when it decreases equally in equal times.

Uniform motion is estimated by the time employed in

moving over a certain space, or in other words, by the space moved over in a certain time.

CASE I.

To ascertain the velocity.

Divide the space run over by the time.

CASE II.

To ascertain the distance.

Multiply the velocity by the time.

In accelerated motion the space run over (or distance) is as the square of the time.

1. A body acted upon only by one force, will always move in a straight line.

2. Bodies acted upon by two single impulses, whether equal or unequal, will also describe a right line.

The *Momentum* of a body is the force with which it moves, and is in proportion to the weight or quantity of matter, multiplied into its velocity.

The action of bodies on each other are always equal, and exert in opposite directions; so that any body acting upon another, loses as much force as it communicates.

CENTRAL FORCES.

The central forces are its *centrifugal* and *centripetal* forces.

The centrifugal force is the tendency which bodies that revolve round a centre, have to fly from it in a tangent to the curve they move in, as a stone from a sling. 2. The centripetal force is that which prevents a body from flying off by impelling it towards the centre, as the attraction of gravitation.

CENTRE OF GRAVITY.

1. The centre of gravity is that point in a body about which all its parts exactly balance each other in every direction.

1. A vertical line passing through the centre of gravity of a body, is called the line of direction.

2. When the line of direction falls within the base of a body, that body cannot descend; but if it falls without the base, the body will fall.

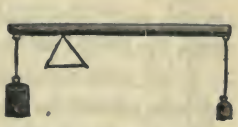
MECHANICAL POWERS.

That body which communicates motion to another is called the *power*.

The body which receives motion from another, is called the *weight*.

The mechanical powers are five the *lever*, the *wheel* or *axle*, the *pully*, the *screw* and the *wedge*, to these, may be added the inclined plane.

OF LEVERS.



There are 3 orders or varieties of levers, wherein the weights, props or moving powers, may be differently applied to the *vectis* or inflexible bar, in order to effect mechanical operations in a convenient manner. A lever is said to be of the first order when the *prop* is between the *weight* and the *power;* of the second order, when the *weight* is between the *prop* and the *power;* of the third order, when the *power* is between the *prop* and the *weight.* A power and weight acting upon the arms of a lever will balance each other, when the distance of the point at which power is applied to the lever from the prop, is to the distance of the point at which weight is applied, as the weight is to the power.

To find what weight may be raised by a given power.

RULE.—As the distance between the body to be raised or balanced, and the fulcrum or prop, is to the distance between the prop and the point, where the power is applied, so is the power to the weight which it will balance.

1. If a lever be 100 inches long, what weight lying 7½

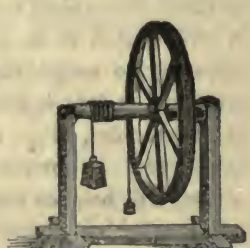
inches from the end, resting on a pavement, may be moved with the force of 168 lbs. lifting at the other end of the lever? Ans. 2072 lbs.

2. A water wheel turns a crank working three pump rods, fixed 6 feet from the joint or pin which their several levers, each 9 feet in length are fastened on account of the intended motion at one end, the suckers of the pumps being put in operation by the other, proves them to be levers of the third order. Now if the crank in this case plays 9 inches round its centre, what is the length of the stroke in each of the barrels? Ans. 27 inches.

3. With what force of water ought that water wheel be driven, (circumstanced as in the last question) which raises 3 cubic feet of water at every revolution of the wheel, each experimentally weighing $62\frac{1}{2}$ lbs. Avoirdupois, the friction of the wheel rejected? Ans. $281\frac{1}{4}$ lbs.

4. At what distance from a weight of 1530 lbs. must a prop be placed, so that a power of 170 pounds, applied 9 feet from the prop, may balance it? Ans. 1 foot.

THE AXLE OR WHEEL.



The wheel or axle is a wheel turning round together with its axis the power is applied to the circumference of the wheel, and the weight to that of the axis, by means of cords, an equilibrium is produced in the wheel and axis, when the wheel is to the power, as the diameter of the wheel to the diameter of its axis.

RULE.—As the diameter of the wheel is to the diameter of the axle, so is the weight to be raised by the axle, to the power that must be applied to the wheel.

CASE I.

1. If the diameter of the axle be 6 inches, and the diameter of the wheel 4 feet, what power must be applied to the wheel to raise 960 lbs. at the axle? Ans. 120 lbs.

CASE II.

2. If the diameter of the axle be 6 inches, and the diameter of the wheel 5 feet, what power must be applied to the axle to raise 300 lbs. at the wheel? Ans. 2400lbs.

CASE III.

3. If the diameter of the axle be 8 inches, and 300 lbs. applied to the wheel to raise 2400 lbs. at the axle, what is the diameter of the wheel? Ans. 64 inches.

CASE IV.

4. If the diameter of the wheel be 64 inches and 300 lbs. applied to the wheel to raise 2400 lbs. at the axle, what is the diameter of the axis? Ans. 8 inches.

PULLEY.

The pulley is a small wheel moveable about its axis by means of a cord which passes over it. When the axis of the pully is fixed, the pully only changes the direction of the power; if moveable pulleys are used an equilibrium is produced when the power is to the weight, as one to the number of ropes applied to them, if each moveable pully has its own rope, each pully will be double the power.

To find the weight that may be raised by a given power.

RULE.—Multiply the power by twice the number of moveable pulleys, and the product is the weight.

CASE I.

1. What power must be applied to a rope that passes over one moveable pully, to balance a weight of 400 lbs.? Ans. 200 lbs.

CASE II.

2. What weight will be balanced by a power of 20 lbs. attached to a cord that passes over three moveable pulleys? Ans. 120 lbs.

CASE III.

3. What weight will be balanced by a power of 100 lbs. attached to a cord, that passes over 2 moveable pulleys? Ans. 400 lbs.

CASE IV.

4. If a cord that passes over two moveable pulleys be attached to an axle 6 inches in diameter, and if the wheel be 60 inches in diameter, what weight may be raised by the pulley, by applying 100 pounds to the wheel?

Ans. 4000 lbs.

INCLINED PLANE.

An inclined plane, is a plane which makes an acute angle with the horizon.

The *motion* of a body descending down an inclined plane is uniformly accelerated. The force with which a body descends by the force of attraction down an inclined plane is to that, with which it would descend freely as the elevation of the plane is to its length, or as the size of its angle of inclination to radius.

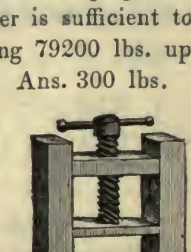
To find the power that will draw a weight of an inclined plane.

RULE.—Multiply the weight by the perpendicular height of the plane, and divide the product by the length.

1. An inclined plane is 60 feet in length, and 15 feet perpendicular height, what power is sufficient to draw up a weight of 1000 lbs. Ans. 250 lbs.

2. A certain Railroad, 1 mile in length has a perpendicular elevation of 20 feet, what power is sufficient to draw a train of baggage cars, weighing 79200 lbs. up this elevation? Ans. 300 lbs.

THE SCREW.



The screw is a cylinder which has either a prominent part, or a hollow line passing round it in a spiral form, so inserted in one of the opposite kind, that it may be raised or depressed at pleasure, with the weight upon its up-

per or suspended beneath its lower surface. In the *screw* the equilibrium will be produced, when the power is to the weight, as the distance between the two contiguous threads, in a direction parallel to the axis of the screw to the circumference of the circle described by the power in one revolution.

To find the power that should be applied to raise a given weight.

RULE.—As the distance between the threads of the screw is to the circumference of the circle described by the power, so is the power to the weight to be raised.

Note.—One-third of the power is lost in overcoming friction.

1. If the threads of a screw be 1 inch apart, and a power of 100 lbs. be applied to the end of a lever 10 feet long, what force will be exerted at the end of the screw?

Ans. 75398.20 + lbs.

THE WEDGE.

The wedge is composed of two inclined planes whose bases are joined. When the resisting forces and the power which acts on the wedge are in equilibrio, the weight will be to the power, as the height of the wedge to a line drawn from the middle of the base to one side, and parallel to the direction in which the resisting force acts on that side.

To find the force of the Wedge.

RULE.—As the breadth or thickness of the head of the wedge is to one of its slanting sides, so is the power which acts against its head to the force produced at its side.

1. Suppose 100 pounds to be applied to the head of a wedge that was 2 inches broad, and 20 inches long, what force would be effected on each side? Ans. 1000 lbs.

ON THE STEAM ENGINE.

Steam at the temperature of 212° is 1800 times its bulk in water; or, one cubic foot of steam, when its elasticity is equal to thirty inches of mercury, contains one cubic inch of water. Therefore, when an engine in good order, is performing its regular work, the effective pressure may be taken at eight pounds on each square inch of the surface of the piston.

CASE I.

To calculate the power of an Engine.

It has been demonstrated in the *Franklin Institute*, of Philadelphia, that a horse can draw 200 lbs. at the rate of 2¼ miles an hour, or 220 feet in a minute with a continuance drawing over a pully, that is, 200 × 220 = 44000 lbs. at 1 foot per minute, or 1 lb. at 44000 feet per minute.

RULE 1.—Multiply the area of the cylinder by the effective pressure, say 8 lbs., the product is the weight the engine can raise. Multiply this weight by the number of feet the piston travels in one minute, the product will give the *momentum*, divide this momentum by a horse power, and the quotient will be the number of horse power in the engine.

2. The velocity of an engine being 220 feet per minute, 25 inches of the area of the cylinder is equal to one horse power.

TABLE FIRST.

Length of stroke.	*Number of strokes.*	*Feet per minute.*
Feet 2	43	172
" 3	32	192
" 4	25	200
" 5	21	210
" 6	19	228
" 7	17	238
" 8	15	240
" 9	14	250

1. What is the power of an engine, the cylinder being 42 inches diameter and stroke 5 feet?

$$\frac{42^2 \times .7854 \times 8 \times 210}{44000} = 74.44 \text{ horse power.}$$

2. What size of cylinder will a 60 horse power engine require when the stroke is 6 feet?

$$\frac{44000 \times 60}{228 \times 8} = 1447.39 + \text{ area of cylinder.}$$

Examples calculated by Rule Second.

1. What diameter is the cylinder of a 40 horse engine, common pressure?

$$\frac{\sqrt{40 \times 25}}{.7854} = 35.7 \text{ say } 35\tfrac{3}{4} \text{ inches diameter.}$$

CASE II.

To find the power to lift a weight at any velocity.

RULE.—Multiply the weight in pounds, by the velocity in feet, and divide by the horse power, the quotient will be the number of horse power.

TABLE SECOND.

When the effective pressure on each inch of piston is	*The area equal to one horse power will be*
53 pounds.	3.70 inches.
48 "	4.17 "
43 "	4.65 "
38 "	5.26 "
33 "	6.06 "
28 "	7.14 "
23 "	8.70 "
18 "	11.11 "
13 "	15.46 "
8 "	25.00 "

1. What diameter is the cylinder of a 40 horse engine, *effective pressure* 33 lbs. on the square inch?

$$\frac{\sqrt{40 \times 6.06}}{.7854} = \text{say } 17\tfrac{5}{8} \text{ inches diameter.}$$

2. The cylinder of an engine is 40 inches diameter, and the effective pressure is 20 lbs. on the square inch, what is the power of the engine?

$$40^2 \times .7854 = 1256.64$$

$$\frac{1256.64}{8} = 157.08 \text{ horse power.}$$

GENERAL THEOREMS.*

THEOREM 1.

When the sum and difference of any two numbers are given, to find the numbers.

RULE.—To half the sum add half the difference, for the greater number. From half the sum, take half the difference for the lesser number.

THEOREM 2.

The product of the sum and difference of any two numbers is equal to the difference of their squares.

THEOREM 3.

If the difference of the squares of two numbers be divided by their difference, the quotient will be the sum; and if by the sum of the numbers, the quotient will be the difference.

THEOREM 4.

If a number be divided into any two parts, the square of the number is equal to the sum of the squares of the two parts, and twice the product of those parts.

THEOREM 5.

If the difference of the cubes of any two numbers be divided by their difference, the quotient arising will be equal to the sum of the squares of the two numbers together with their product.

THEOREM 6.

When the sum and product of two numbers are given to find the numbers

* These Theorems should be committed to memory.

RULE.—Square the sum of the numbers, and from that square, subtract 4 times the product, the square root of that difference will give the difference of the numbers, then agreeably to Theorem 1, by having the sum and difference we can easily find the numbers.

THEOREM 7.

The sum, and sum of the squares of two numbers being given to find the numbers.

RULE.—Subtract the sum of the squares of the numbers, from the square of their sum, and half the remainder will be their product. When the sum and product are given, we can find the numbers agreeably to Theorem 6.

QUESTIONS FOR EXERCISE.

1. A wealthy man two daughters had,
   And both were very fair,
   To each he gave a tract of land,
   One *round*, the other *square*.
   At twenty shillings an acre just,
   Each piece its value had;
   The shillings that did compass each
   For it exactly paid.
   If 'cross a shilling be an inch,
   As it is very near,
   Which was the greater fortune,
   She that had the round or square?

Ans. Area of the square = 25090560 acres.
Area of the circle = 19706125.8240 acres.

A. A bullet is dropped from the top of a building, and found to reach the ground in 2 seconds. Required its height? Ans. $2 \times 4 = 8$ and $8 \times 8 = 64$ feet.

B. Admitting I let fall a bullet from the top of a building, and found it reached the ground in $1\frac{1}{2}$ seconds. Required its height? Ans. 36 ft.

2. If a roll of butter weighs in one scale $2\frac{1}{4}$ lbs., and being changed into the other weighs 4 lbs.; what is the true weight of the butter? Ans. 3 lbs.

3. A farmer hired two men, Reuben and Richard, for $30, (being the wages of both for 1 month,) to Reuben he gave $4 per month more than to Richard. How much did each get per month?

Ans. Reuben got $17 and Richard $13.

4. There is a certain number which being divided by 11, the quotient resulting multiplied by 5, that product divided by 4, from the quotient subtract 75, to the remainder add 40, and half the sum shall make 45. Required the number? Ans. 1100.

5. What is the discount of $400 for 2 years, 8 months, 24 days, at 6 per cent. per annum? Ans. $56.36.

6. What is the interest of $400 for 2 years, 8 months, 24 days, at 6 per cent.? Ans. $65.60.

7. A merchant in Baltimore received from New Orleans a bill at 30 days sight; he allowed 1 per cent. discount for present payment, and received $2530.44; what sum was the bill drawn for; and what was the discount?

Ans $2555.74 + disct. $25.30.

8. Bunker Hill monument is 30 feet square at its base, and is to be 15 feet square at its top, its height is to be 220, from the bottom to the top through its centre, is a cylindrical avenue 15 feet in diameter at the bottom, and about 11 feet at the top. How many cubic feet will there be in the monument? Ans. 86068.518 + ft.

9. Divide 40 into 4 such parts, that if to the first you add 4, from the second subtract 4, multiply the third by 4, and divide the fourth by 4; the sum, difference, product and quotient will be equal? Ans. $\frac{1}{2}^{2}$, $\frac{5}{5}^{2}$, $\frac{2}{5}$ and $1\frac{2}{5}^{8}$.

10. Find two numbers in a given ratio as 6 to 9, so that their sum and product may be equal? $2\frac{1}{2}$ and $1\frac{1}{2}$.

11. Two drovers, A and B, (from the state of Ohio) stopped at the "Three Tun Tavern," in the city of Baltimore, each having a drove of cattle; A had 21 head, in his drove, and B 19; they agreed to join both together, and sell them for $27 per head; and, that A should receive

$3 a head more than B; how much did each get for his cattle? Ans. A got $596.92½; B $483.07½.

12. What number is it to which ¼ of ⅘ of ⅞ be added, the sum will be 1? Ans. 3⅓.

13. What number is it whose ½, ⅓ and ¼ added together make 78? Ans. 72.

14. To find a number, which being multiplied by 3, subtracting 5 from the product and the remainder divided by 2, if the number sought be added to the quotient, the sum will be 40? Ans. 17.

15. What number is it which being added to 4, and also multiplied by 4, the product shall be treble the sum? Ans. 12.

16. To find a number, to which if 11 be added and 7 subtracted from the same number, the sum of the addition will be double the remainder? Ans. 25.

17. To find a number which being added to itself and the sum being multiplied by the same, and the same number still subtracted from the product; and lastly, the remainder divided by the same, the quotient may be 13? Ans. 7.

18. To find three numbers such, that the sum of the first and second shall be 15, the sum of the first and third 16, and the sum of the second and third 17? Ans. 7, 8 and 9.

19. Three persons, A, B and C, owe certain sums of money, so that A and B owe $210, B and C 290, and C and A 400; what did each of them owe? Ans. A 160, B 50, and C 240.

20. A man having a certain number of dollars in his hand, being asked how many he had, replied if the number be divided by 5 and 19 added to the quotient, I shall then have $23; how many dollars had he? Ans. 20.

21. A person said he had 20 children, and that it happened there was a year and a half between each of their ages, his eldest was born when he was 24 years old, and

the age of the youngest is now 21; what was the fathers' age? Ans. 73½ years.

22. When I sold cloth at $7 per yard, I gained 56¼ cents; what would I gain by selling 3 pieces, which cost me $400? Ans. $32.14¼.

23. A and B dissolve partnership, and equally divide their gain; A's share which was $332.50 lay for 21 months; B's for 9 months only; the adventure of B is required? Ans. $775.83⅓.

24. I bought 25 yards of lace at 30 cents for the first yard, and for the last yard 96; the price of each yard increasing in arithmetical progression; what did the whole amount to? Ans. $15.75.

25. A case of dry goods amounting to £230 5*s*. sterling is sold in Philadelphia, at 20 per cent. advance; what is the amount in American currency? Ans. $1228.

26. What is the difference between the interest of $1000 at 6 per cent. for 8 years, and the discount of the same sum at the same rate, and for the same time?

Ans. the interest exceeds the discount by $155.67½.

27. A gentleman sells a piece of cloth at $56.25, and thereby loses 7½ per cent.; what was the first cost? Ans. $60.47.

28. A has 28 cwt. of cotton which cost him $264; how must he rate it per lb., to D, so that by taking his note payable at 9 months he may clear $30, allowing interest at 6 per cent. per annum? Ans. 9¾c.

29. In a series of proportional numbers the first is 5, the second is 8, the product of the second and third is 78.4, what is the difference of the first and fourth? Ans. 10.68.

30. If 3481 soldiers are to be placed in square file, how many are to be set in each rank? Ans. 59.

31. A can do a piece of work in 10 days, B in 13, set them both at it, in what time will it be finished? Ans. 5$\frac{5}{23}$.

32. A stationer sold quills at $10 per thousand, by which he gained $\frac{3}{8}$ of the money, but growing scarce raised them to $12,50 per thousand, what is the gain per cent. by the latter price? Ans. 100 per ct.

33. Richard Cotter, of Baltimore, remits to William Denman, of New York, a bill of exchange on London, the avails of which he wishes to be invested in goods on his account. Denman disposes of the bill at 7½ per cent. advance, and received $9675.00, having reserved for himself ¼ per cent. on the sale of the bill, and 2 per cent. for commission, what will remain for investment, and how much was the bill drawn for?

Ans. for investment $9457.31¼. The bill was drawn for £2025 sterling.

SOUND.

Sound if not interrupted, will move at the rate of about 1150 feet in a second of time.

34. I have seen the flash of a cannon fired from Fort McHenry, and heard the report 47 seconds afterwards, what distance was the fort from where I stood?

Ans. 54050 feet.

35. Bought a quantity of cloth for $750, ¼ of which I found to be inferior, which I had to sell at $1.25 per yard, and by this I lost $100, what must I sell the rest at per yard, that I shall lose nothing by the whole?

Ans. 3.15\frac{1}{7}$.

36. A circular fish pond is to be dug in a garden that shall take up just half an acre, what must be the radius of the circle? Ans. 27.75 yds.

37. Bought a horse which was worth 30 per cent. more than I gave for him, but having been injured I sold him for 25 per cent. less than what he cost, and thereby lost $55 on his real value, what was received for the horse?

Ans. $75.

38. A son having asked his father's age the father thus replied: "your age is 8 years, to which if five-eights of

both our ages be added, the sum will be my age," what was the father's age? Ans. 34 years, 8 months.

39. A merchant sold goods to a certain amount on a commission of 4 per cent., and having remitted the net proceeds to his correspondent, he received ¼ per cent. for prompt payment, which amounted to $15.60, what was the amount of his commission? Ans. $260.00.

40. How many thousand brick would be required to build the walls of a house 40 feet long, 30 feet wide, and 20 feet high, admitting the walls to be a foot thick, and that each brick was 8 inches long, 4 inches wide, and 2 inches thick? Ans. 73440.

41. Purchased merchandise to the amount of $2000, viz: $400 at 3 months, $800 at 6 months, and $800 at 9 months. Required the average time of payment.

Ans. 6⅔ months.

42. Bought several parcels of goods at different times and on various credits.

1841.

May 11,	a bill am'ting	to	$75	at	1	mo.	crd.	due June 10.
" 18,	"	"	64	"	2	"	"	" July 17.
" 25,	"	"	96	"	3	"	"	" Aug. 23.
June 6,	"	"	104	"	4	"	"	" Oct. 4.
" 20,	"	"	144	"	5	"	"	" Nov. 17.

Required the average time of payment?

Ans. July 14th, at 9 o'clock in the morning.

43. The ball on the top of St. Paul's Church is 6 feet in diameter, what did the gilding of it cost at 3½d. per square inch? Ans. £237 10s. 1d.

44. There is a conical glass, 6 inches high, 5 inches wide at the top, and is ½ part filled with water, what must be the diameter of a ball, let fall into the water, that shall be immersed by it? Ans. 2.445 + in.

45. How much larger is a cube, that will contain a globe of 20 inches in diameter, than a cube inscribed within such globe? Ans. lar. 8000 in. smaller 1539 in.

SUBSCRIBERS' NAMES.

CLERGYMEN.

Rev. G. Raymond, D. D. President St. Mary's College
" E. A. Knight, V. Pres't
" L. R. Deluol
" Hugh Griffin
" F. Lhomme
P. Courtney
" E. Damphoux, St. Joseph's Church
" John B. Gildea, St. James'
" P. S. Schreiber, Cathedral
" Chas. J. White, "
" Joseph Prost, St. John's
" N. Kerney, St. Patrick's Fell's Point
" James Dolan, " "
" David Steele, Exeter st.
" John F. Hey, Fayette st.
" John Smith, Pastor of Seamen's Bethel, F. P.
" Samuel Kepler, Wilk st.
" Gerard Morgan, Car. st.
" John A. Gere, Aisquith st.
" John Valiant, Wilk st.
" John Watts, Lexington st.
" John Russel, (German Evangelical)
" Henry Scheib (German Lutheran Zions' Church)

TEACHERS.

John Neely, A. B. Hanover st.
John Magee, Esq. Fred'k st.
Hugh Bannan "
Edmund Smith "
David Ring, E. B. F. Institute
Patrick H. M'Sorley
N. Spelman
Robert S. Wrixon
J. J. Reekers, prof. of M. & L.
Jos. Walker, prof. of Math.
J. N. M'Jilton, Esq. 20 Eutaw st.
Frederick Foy
James E. Searly
Henry Haupt
James McCrodan
Shepard C. Keith
James McCarrick
James McIntire, A. M.
James Harshaw, A. M.
John Galvin, M. C. T.
George Sharp
Richard Cotter, Esq. Professor of Mathematics
Z. George, prof. of Languages
John H. Shea
Henry McCann
Edward Hazen, Author
S. H. Matthews
N. C. Brooks, A. M. Principal *High School in Baltimore.*
P. Kelly, professor St. Mary's Seminary

LAWYERS.

Hon. R. B. Taney, C. J. U. S.
Hon. Sol. Hillen, M. C.
James M. Buchanan
Wm. P. Preston
Wm. H. Watson
Wm. C. Collins
John C. Legrand
Chas. H. Pitts
Geo. R. Richardson
T. Parkin Scott
Wm. Gwynn
Chas. Z. Lucas
Samuel Stump, jr.
William F. Giles
John Scott

PHYSICIANS.

H. A. Inloes
John C. S. Monkur
John F. Monmonier
Nathaniel Potter
P. S. Kinneman
Henry Morris
Charles Maguire
Samuel G. Baker
Nathan R. Smith
Richard Wilmot Hall
Levi D. Bodder
Jas. McIntire
William E. A. Aikin
Daniel H. Lawrence
D. P. Chatard
John H. O'Donovan
Andrew A. Lynch
G. Schwarz

H. W. Waters (Mana Grove)
T. L. Murphy (Bond st. F. P.)

MAGISTRATES.

Jeremiah Storm
Simon Kemp
Geo. W. King
Wm. Pechin
Wm. B. Jones
John Burnham
John Wright
M. Bush
J. Walsh
James Blair
Peregrine Gorsuch
Jas. Barnard, jr.
Jas. P. Heath
Henry R. Lauderman
A. H. Pennington
Chas. Keenan
Henry Snyder
John Walter
Eugene O'Connor

NOTARIES.

Robert M. Welch
William Eichelberger

PRINTERS.

Wm. G. Warner
Richard J. Matchett
Jno. Murphy, 146 Market st.
Sherwood & Co.
James Young
A. H. Simmons, Ed. "Sun"

BOOK SELLERS.

David Owen, 24 N. Gay st.
Geo. P. Knotts, Market st. F. P.
Knight & Colburn

MERCHANTS.

Meredith, Spencer & Co., 246 Baltimore st.
P. B. Curtiss, Han. & Lomb.
J. W. Randolph, Market st. F P.
Wm. Jackson, Thames st.
Wm. Wilson & Sons, 105 Market st.
T. J. Ahern, Liberty & Balt. sts.
Elijah Stansbury
Francis Hyde & Sons
A. Penniman, North st.
John O'Brien
Daniel Coonan
Benjamin Thomas
James Dickson
B J & E J Saunders
James Hooper, Balt. st.
Geo. Waters & Son, F. P.
Wm. Gibson,
Lewis Auodoun
Ezekiel Dorsey
M. Stillinger
Isaac Hartman
Patrick Mullin
Wm. G. Bennett
John Bennett
John Thompson
Samuel Wilson
Lorenzo D. Taylor
Wm. A. Sloan
Thos. J. Warrington
Edwd. W. Bennett
Turner, Wheelwright & Mudge
John J. Smith, accountant
O'Connor & Co., North st.
M. Blundell, 73 N. Howard st.
Wm. Wilson, 16 Franklin st.
E. McFaul & Co., Paca st.

MERCHANTS AND GROCERS.

Thos. R. Simmons
Alfred Wiles
Washington Brown
G. M. Hutton, 24 Baltimore st.
John T. Laws, accountant
Robert Barry, Esq.
Lemuel A. Gordon
Robert Clark
John I. Gross, auctioneer
George Pressman
Thos. E. Reardon
Edwd. W. Bennett
Thomas Walsh, 18 S. Gay st.
Alexander Ager, Second st.
John T. Burns, accountant
Thomas P. Jenkins, Gay st.
David C. Springer, Pratt st.
George H. Keys, Gough st.
John T. Spencer
James A. Morris
John Drum
Justus Bruehl
Edward P. Brennan
John Bates
B. McDonough
John Matthews, 59 S. Charles st.
Jeremiah O'Brien
O. F. Pettit
James M'Court
Wm. H. Fowler
Wm. Wardenburg
Samuel P. Curry
Mich'l McGrath, Eutaw & Mad.
Frederick Eversman

Thos. Murdock
Thos. Flaherty
William Caldwell
John G. Tebelman
P. Collins
William Solomon
John W. Croney
Mark Hagan
Stephen Abbott
R. Stansbury
B. Curry
Martin Bragtigan
John Ball
John Saurhoff
Myer Hart
Moses Wiesenfeld
Thomas Leib, jr.
Samuel Swobe
Philip Fell
Emanuel Weinman
Joseph Zinkand
Leonard Shuman
Robt. Thompson
James Diggs
Anthony Keenan, Union Hotel
Edwd. Boyle, 9th Ward House
P. Adams, Hotel, 2 Light st.
James Brown, Washington hotel
Michael McCaraher, 'Barnum's'

APOTHECARIES.

Thos. J. Pitt
James P. Williamson
Joseph B. Stansbury

MUNICIPAL OFFICERS.

Hon. S. C. Leakin
Hon. Samuel Brady, Mayor
John Mitchell, H. C.
H. G. Miller
M. Benzinger, Esq.
Jacob Small, Esq.
Thos. Stansbury

CITY COUNCIL.

John E. Stansbury
Joseph Ramsay
James Fields

CITY OFFICERS.

Jeremiah Frazier
Albert C. Hall
William A. Swift
Edward C. Taylor

SEA CAPTAINS AND OFFICERS U. S. SERVICE.

Wm. S. Coupland, E. Pratt st.
Joshua Ferrell
Michael McDonald
John M'Gowan, U. S. R. S.
James Frazier
W. W. Stafford
John Besse, U. S. R. S.
Chas. B. Beaufort, "
Robert Benthall
Gay Moore, U. S. R. S.
Lieut. Wm. Wall, U. S. A.
H. Prince "
H. Davey
Wm. Thompson
James Gibson
Joseph Ennis
Wm. McLennan
W. S. Wedge
James Gunby
Wm. C. Lewis
R. Robinson
Thos. T. Wingale
Edwin Bailey
Henry H. Willis
Lieut. Sands, U. S. R. S.
" Beverly Diggs "
" A. Burrough "
J. J. B. Walbach, U. S. N.
J. M. Gardner, " "
James Kelly, U. S. A.
COMMODORE DANIELS
C. H. Cooksey
James Beard
J. B, Corner
Samuel Keene
Samuel Wall
John H. Fry
John S. Oliver
James Grieves
John Dutton
Thomas Kelly
Edward Gould
Wm. B. Higgins
John Myers
Jacob W. Hugg
George Little
Walter Oram
John Martin

PILOTS.

Hector Jackson
Robert M. Bear
James Cheveral
Thos. G. L. Clark
James A. Milbourn
J. F. Haney
William Chase
Thomas Lewis
John Cost

HATTERS.

Chas. Townsend
Levi Lowe
P. Murray

ENGRAVERS.

J. Madison Anderson
J. S. Horton
Geo. W. Taylor, North st. No. 8

PAINTERS.

Richard R. Bishop, Thames st.
M. O'Laughlin, cor. Holliday and Fayette st.
John Dobson, Wolf st. F. P.
John Welch, Ann st. "
Duvall & Maccubbin, Fayette st.
Wm. Johnson, 73 Lombard st.

PLUMBERS & COPPERSMITHS.

Nathan H. Hall
Henry Boss, Thames st.
James Kelly, 53 South st.
John M. Bruce, Light st.
Benj. F. Barkman, Thames st.
Edward G. Dorry, Thames st.
Frederick Daiger, Pratt. st.
Francis Kines, Light st.

COOPERS.

Nicholas Gill, near Eutaw house
Wm. S. Espy & Co., 27 Ann st.
Jos. Golibart, sr., Alice Ann st.
Jos. M'Dermott, Philpot st.
Benj. Abbott, President st.
Alex. Logan, Lerew's alley
Thos. Woollen, Commerce st.
John K. Carroll, "
Martin Corban "
Jos. Golibart, jr., Ann st.
Henry Hush, Howard st.
Robert Wilson, Fayette st.
Reuben Hicks, German st.
J. W. Allen, German near Paca
George H. Watts, 272 Market st.
Henry Cook, Lee st.
Wm. Patterson, "
Richard Pierce
James Pouch
Wm. Douglass, Green st.

COACH MAKERS.

James McCurley, Liberty st.
Wm. Selvage, Fayette st.
James Stewart, Calvert st
Robert Leckie, Philad. Road
Wm. McCann, Frederick st.
Thomas C. Dunlevy, Gay st.

CUSTOM HOUSE OFFICERS.

Col. Wm. Frick, collector
J. K. Law, Esq., dep. "
R. Lyon, jr.
J. W. Wilmer, 7 Bowly's wharf
Henry Wigart
Charles Soran
Wm. V. Jenkins
George W. Burke, Fountain row
Richard Mackubbin "
Thomas Kyle, F. P.
D. W. Hudson, 10 Fountain row
John Nants, 72 High st.
Robert Aitken, 191 E. Pratt st.
James Jenkins, Lovely Lane
John Lowry, Mechanics' row, Wilk st.

PROPERTY AGENTS.

Jas. Tracey, St. Paul st.
Geo. Gelbach, broker, N.W. cor. Baltimore and Frederick st.
Abm. Cuyk, 42 Fell st., F. P.
Chas. Dieselhorst, 24 Thames st.

WATCHMAKERS.

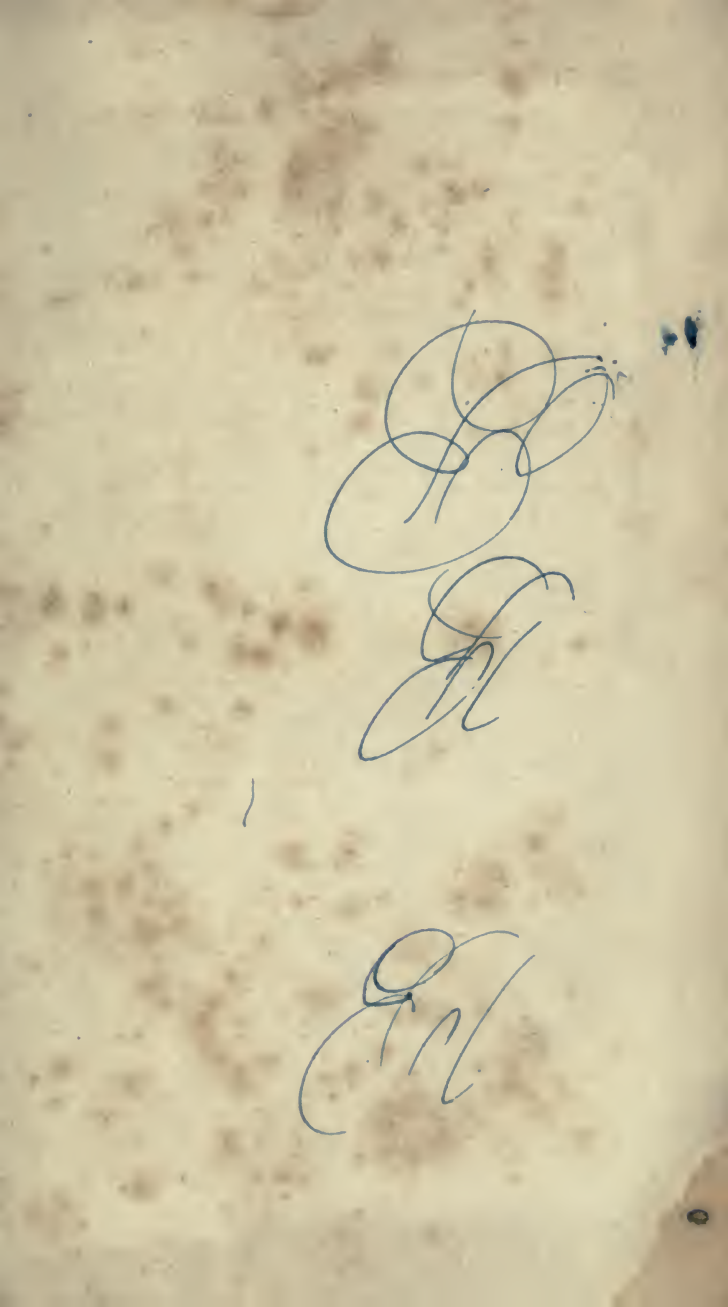
Geo. A Syberts, Bond st.
John Somer "
Peter Riefner, "
Aaron Hart, "
Jas. V. Brown, "
A. Ostendorff, Camden st.
Everhard Krimmer
Robert Holloway, High st.

CABINET MAKERS.

Edwin S. Tarr, 4 Gay st.
Wesley B. Tarr, 32 "
R. A. L. Bevans, E. Balto. st.
John Hughes, Bond st.
John Garrett, Alice Ann st.
Jas. P. Weakley, Pitt st.
Jas. Byrne, Fred. & Lom. st.
Jacob Rulf, Caroline st.
George Hoffman, Park st.
Wm. King, 72 Charles st.

CARPENTERS.

John Roy, Hanover st.
Thos. C. Monmonier, F. P.
Wm. Campbell, Washington st.
Geo. A. Heuisler, Madison st.
W. E. Beale, Silver & Spring st.
E. W. Robinson, Market st. F. P.
John R. Budd, Alice Ann st.
John M'Ardle, York avenue
Mich. Flaherty, Washington st.
H. W. S. Evans, cor. Gough & Eden st.



YB 35827

M577035

QA102
M17
Educ.
Lib.